ELASTIC AND ACOUSTIC WAVEFIELD DECOMPOSITIONS AND APPLICATION TO REVERSE TIME MIGRATIONS

by

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Dedicated to my family and teachers
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by

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P- and S-waves coexist in elastic wavefields, and separation between them is an essential step in elastic reverse-time migrations (RTMs). Unlike the traditional separation methods that use curl and divergence operators, which do not preserve the wavefield vector component information, we propose and compare two vector decomposition methods, which preserve the same vector components that exist in the input elastic wavefield. The amplitude and phase information is automatically preserved, so no amplitude or phase corrections are required.

The decoupled propagation method is extended from elastic to viscoelastic wavefields

To use the decomposed P and S vector wavefields and generate PP and PS images, we create a new 2D migration context for isotropic, elastic RTM which includes PS vector decomposition; the propagation directions of both incident and reflected P- and S-waves are calculated directly from the stress and particle velocity definitions of the decomposed P- and S-wave Poynting vectors. Then an excitation-amplitude image condition that scales the receiver wavelet by the source vector magnitude produces angle-dependent images of PP and PS reflection coefficients with the correct polarities, polarization, and amplitudes.
It thus simplifies the process of obtaining PP and PS angle-domain common-image gathers (ADCIGs); it is less effort to generate ADCIGs from vector data than from scalar data.

Besides P- and S-waves decomposition, separations of up- and down-going waves are also a part of processing of multi-component recorded data and propagating wavefields. A complex trace based up/down separation approach is extended from acoustic to elastic, and combined with P- and S-wave decomposition by decoupled propagation. This eliminates the need for a Fourier transform over time, thereby significantly reducing the storage cost and improving computational efficiency. Wavefield decomposition is applied to both synthetic elastic VSP data and propagating wavefield snapshots. Poynting vectors obtained from the particle-velocity and stress fields after P/S and up/down decompositions are much more accurate than those without.

The up/down separation algorithm is also applicable in acoustic RTMs, where both (forward-time extrapolated) source and (reverse-time extrapolated) receiver wavefields are decomposed into up-going and down-going parts. Together with the crosscorrelation imaging condition, four images (down-up, up-down, up-up and down-down) are generated, which facilitate the analysis of artifacts and the imaging ability of the four images. Artifacts may exist in all the decomposed images, but their positions and types are different. The causes of artifacts in different images are explained and illustrated with sketches and numerical tests.
# TABLE OF CONTENTS

ACKNOWLEDGMENTS ................................................................. v
ABSTRACT ........................................................................ vi
LIST OF FIGURES ................................................................. xii
LIST OF TABLES ................................................................. xix

CHAPTER 1 INTRODUCTION ......................................................... 1
  1.1 Motivation and objectives .................................................. 1
  1.2 Overview ................................................................. 2
  1.3 Publication status ........................................................ 4
  1.4 Declaration ............................................................... 5

CHAPTER 2 COMPARISON OF DECOMPOSITION METHODS .......... 6
  2.1 Abstract ................................................................. 7
  2.2 Introduction ............................................................. 7
  2.3 Methodology ............................................................ 9
    2.3.1 Selective Attenuation ............................................ 11
    2.3.2 Decoupled Propagation ........................................ 15
  2.4 Test with synthetic Data .................................................. 16
    2.4.1 Decomposition by selective attenuation ..................... 17
    2.4.2 Decomposition by decoupled propagation ................ 20
  2.5 Evaluation of algorithm performance ............................... 20
    2.5.1 Accuracy ......................................................... 20
    2.5.2 Speed ............................................................. 26
    2.5.3 Memory Requirement ........................................... 26
    2.5.4 Numerical Stability ............................................. 27
    2.5.5 Comparison using Marmousi2 data ......................... 28
  2.6 P and S decomposition in x-t gather data .......................... 30
5.2 Introduction ................................................. 85
  5.2.1 Up/down separation ........................................ 86
  5.2.2 P/S + up/down decompositions ............................. 87
5.3 Methodology .................................................. 88
  5.3.1 Previous up/down separations (ω-k domain separation) .......... 88
  5.3.2 Up/down separation using complex traces ...................... 90
  5.3.3 Algorithm implementations and applications ...................... 92
5.4 Poynting vector decomposition .................................. 99
5.5 Elastic RTM with P/S and up/down decompositions ................. 101
  5.5.1 Elastic RTM and ADCIGs without wavefield decompositions .... 103
  5.5.2 Elastic RTM and ADCIGs with P/S decomposition ............... 105
  5.5.3 Elastic RTM and ADCIGs with both P/S and up/down decompositions106
5.6 Discussion .................................................. 108
5.7 Conclusions ................................................. 113
5.8 Acknowledgments ............................................. 113
5.9 Appendix A: Generating discrete-time analytic signals ............ 114
5.10 Appendix B: Calculations of Poynting vectors from Decomposed P- and S-waves115

CHAPTER 6 DECOMPOSED RTM IMAGE ANALYSIS .......................... 117
  6.1 Abstract .................................................. 118
  6.2 Introduction ................................................. 118
  6.3 Methodology ................................................. 121
    6.3.1 Up and down separation using complex traces ................... 121
    6.3.2 Decomposed crosscorrelation imaging condition ............... 122
  6.4 Artifact analysis ........................................... 123
  6.5 Tests on synthetic data ....................................... 124
    6.5.1 Numerical results for a two-reflector model .................... 126
    6.5.2 Numerical results for the Sigsbee model ....................... 130
  6.6 Discussion .................................................. 132
  6.7 Conclusions ................................................. 134
LIST OF FIGURES

2.1  P-wave $Q^{-1}$ as a function of frequency with one relaxation mechanism (the solid line) and the frequency spectrum of the source (the dashed line). .......................... 13

2.2  (a) Snapshots of 2D wave propagation in an elastic homogeneous medium with a composite P and S source at the center. Vertical and horizontal components of the wavefields are recorded. (b) Separation results with the divergence (upper panels) and the curl (lower panels) operators. Note the large differences in the relative amplitudes between (a) and (b), because they are not the same physical waves. Amplitudes along the dashed lines here, and in Figures 2.3 and 2.4 are shown in Figure 2.5. ................................................................. 18

2.3  Snapshots of wave propagation in a viscoelastic medium with strong P-wave attenuation. (a) Decomposed P-waves; (b) decomposed S-waves. Compare with Figures 2.2 and 2.4. ................................................................. 19

2.4  Decomposed snapshots of wave propagation using decoupled propagation. (a) Decomposed P-wavefield; (b) decomposed S-wavefield. Compare with Figures 2.2 and 2.3. ................................................................. 21

2.5  Comparison of waveforms along the dashed lines in Figure 2.2a, 2.3 and 2.4 at 0.5 s. (a) and (b) are horizontal and vertical components of the elastic wavefield (without vector decomposition) from Figure 2.2a. (c) and (d) contain the P-waves, and (e) and (f) contain the S-waves, both decomposed using selective attenuation (the solid blue lines) and the pure P and S waves (the dashed red lines), respectively. (g) and (h) contain the P-waves, and (i) and (j) contain the S-waves, both decomposed using decoupled propagation (the solid blue lines) and the pure P and S waves (the dashed red lines), respectively. .................. 23

2.6  The dotted line shows the residuals using the selective attenuation (of P-waves). The solid line shows the residuals using decoupled propagation. .............................. 24

2.7  (a) True and (b) smoothed velocity model for elastic wavefield extrapolation and P and S decomposition. The red dots are rotational sources. In (a) the upper layer has P- and S-velocity of 1.70 km/s and 1.20 km/s, and the lower layer has P- and S-velocity of 1.90 km/s and 1.40 km/s. .............................. 24
2.8 Snapshots at 0.5s of elastic wavefield and P and S decomposition results in a model with one horizontal reflector at $z = 2.6$ km (a), and the corresponding smoothed model (b). The models are shown in Figure 2.7. In both (a) and (b), the leftmost pair contains the vertical and horizontal components of the elastic wavefield (without decomposition), and the next two pairs, contain the decomposed P- and S-waves using selective attenuation. The two rightmost pairs, contain the decomposed P- and S-waves using decoupled propagation. The sources for both decompositions are rotational, and generate only S-waves. Conversions to P-waves occur at the reflector if using the true velocity model. Both selective attenuation and decoupled propagation decompositions produce artifacts (in the dashed ovals) at the reflector in (a), because of the coupling of P- and S-waves and the generation of head waves, but these artifacts are much reduced when using the smoothed velocity model, in (b).

2.9 Time efficiency comparison for three grid sizes.

2.10 The ratio of wave velocities as a function of frequency in a homogeneous viscoelastic medium $[V(f)]$ and in the elastic medium $(V)$ with the same relaxed (elastic) modulus $M_{1r}$.

2.11 A subset of the elastic Marmousi2 P-wave velocity model; the red circle is the source location used for the $x$-$t$ gather decomposition example, and the blue squares are every 40th receiver.

2.12 Snapshots of the wave propagation in a portion of the elastic Marmousi2 model in Figure 2.11 at time = 0.75 s, using the true P-wave velocity model as the background with wavefield snapshots overlapped with 50% transparency. All snapshots are particle-velocity components; the left column is $v_x$ and the right column is $v_z$. (a) is the complete elastic particle-velocity wavefield without P and S decomposition; (b) is the decomposed P-wavefield using selective attenuation; (c) is the decomposed S-wavefield using selective attenuation; (d) is the decomposed P-wavefield using decoupled propagation; and (e) is the decomposed S-wavefield using decoupled propagation.

2.13 (a) is the recorded seismogram generated from the Marmousi2 model with the direct waves removed; (b) is the decomposed P-wave seismogram; (c) is the decomposed S-wave seismogram. In each pair, the left panel is the horizontal component and the right panel is the vertical component.

3.1 $1/Q_p$ and $1/Q_s$ as functions of frequency. Note that both $1/Q_p$ and $1/Q_s$ are fairly constant within the seismic frequency bandwidth.

3.2 a) Snapshots of 2D wave propagation in a viscoelastic homogeneous medium with a composite P-S source at the center. Horizontal and vertical components of the wavefields are recorded. b) Separation results with the divergence (upper panel) and the curl (lower panel) operators. Note the large differences in amplitudes between (a) and (b).
3.3 Separated snapshots of wave propagation using viscoelastic decoupled propagation. (a) decomposed vector P-wavefield; (b) decomposed vector S-wavefield. 47

3.4 Snapshots of the wave propagation in a part of the elastic Marmousi2 model, using the smoothed P-wave velocity model as the background with wavefield snapshots overlapped with 50% transparency. The $Q_p$ and $Q_s$ values are the $V_p$ and $V_s$ values multiplied by 30 respectively. All snapshots are particle velocity components; the left column is $v_x$ and the right column is $v_z$. (a) is the complete elastic particle velocity wavefield without P and S decomposition; (b) is the decomposed P-wavefield components; (c) is the decomposed S-wavefield components. 48

4.1 Flowchart of elastic RTM with the vector-based prestack image condition. The source wavefield extrapolation is done before the receiver wavefield extrapolation, and both extrapolations include P and S wavefield decompositions in the vector domain. The decomposed P- and S-wave particle-velocity and stress vectors are used to obtain their propagation directions via Poynting vectors. Because of the different polarizations of PP and PS reflections, the procedures for determining reflection signs in the image condition are also different. 55

4.2 (a) Horizontal and (b) vertical particle-velocity components of an elastic wavefield from P- and S-wave sources at different locations contain superimposed P- and S-wave fronts. 61

4.3 Enlarged image of the vector sum of the particle-velocity components in the boxed area in Figure 4.2a and 4.2b. The arrows indicate the particle-velocity vector directions in the superimposed P and S wavefronts in (a), and the corresponding Poynting vectors in (b), which give the energy flow directions of the superimposed P- and S-waves. After P and S vector wavefield decomposition, the particle velocity polarization directions (in c and e) along with the stress vectors produce Poynting vector directions in (d) for the decomposed P-waves and in (f) for the decomposed S-waves. The Poynting vectors are calculated by equations 5.10 and 5.11, for the P- and S-waves, respectively. 62

4.4 Kinematic example of (a) PP and (b) PS particle motions and the corresponding reflection signs. $\tilde{S}_i^{src}$ and $\tilde{s}_i^{rec}$ are propagation directions of the vector source (src) and receiver (rec) wavefields, where $i$ is the wave mode (P or S); $n$ is reflector normal determined from the incident and reflected P-waves; $V_i^{src}$ and $\tilde{v}_i^{rec}$ are particle-velocity vectors; $\dot{V}_i^{P,src}$ and $\tilde{v}_i^{P,rec}$ are incident and reflected P-wave particle-velocity vectors projected onto the reflector normal; $\dot{V}_i^{P,src}$ and $\tilde{v}_i^{S,rec}$ are the projections, of the incident P-wave and reflected S-wave particle-velocity vectors, tangent to the reflector. 67

4.5 (a) Three-reflector model; the red spot is the source location, and the blue squares are every 35th receiver. (b) and (c) are the horizontal and vertical particle-velocity seismograms, respectively, with the direct waves removed. Both components contain both P- and S-waves. 71
4.6 (a) is the image time map; (b), (c) and (d) are the P-wave stress map, and the horizontal and vertical particle-velocity component maps, respectively; all correspond to the maximum magnitude of source P-wavefield particle-velocity vector over all times at each grid point. Since the model is smoothed, the P-wave stress values are all at nearly the same phase (the peak or trough); as a result, the values of the P-wave stress have the same sign at all grid points.

4.7 Migrated (a) PP and (b) PS images using the vector-based prestack image condition; (c) and (d) are PP and PS images using normalized cross correlation, and using divergence and curl for P and S separation (with compensation applied). The image resolution of the vector image condition is higher because only the peak values of the source wavefield is used rather than the whole wavelet as in the normalized cross correlation image condition. For both image conditions, the PS images (b and d) have a higher resolution than the PP images (a and c), because the S-wave velocity is lower. See Figure 4.8 for a quantitative amplitude comparison.

4.8 Comparison of peak PP and PS image values from Figure 4.7a and 4.7b for the three reflectors in Figure 4.5a, obtained by using the vector-based (VB) image condition (the red lines), and in Figure 4.7c and 4.7d using the normalized cross-correlation (NC) image condition (the black lines). The PP and PS image values of the uppermost reflector [in panels (a) and (b)] are also compared with values from the Zoeppritz equations [the blue lines in panels (a) and (b)]. All are displayed as a function of surface offset. The critical PP reflection angle (74°) of the uppermost reflector corresponds to surface offset of 5.2 km which is beyond the range of the data plotted here.

4.9 (a) P-wave velocity, (b) S-wave velocity and (c) density distributions used for isotropic elastic wavefield modeling. This is a portion of the Marmousi2 model.

4.10 Sample elastic data simulated for the model in Figure 4.9 with source at x = 2.5 km and z = 0 km, and receivers at z = 0 km: (a) is the horizontal component and (b) is the vertical component of particle-velocity. The direct waves are removed.

4.11 Smoothed (a) P-wave velocity, (b) S-wave velocity and (c) density distributions used for the elastic prestack RTM results in Figures 4.12 and 4.13.

4.12 Representative (a) PP and (b) PS ADCIGs at three horizontal positions of x = 1.5, 3.0 and 4.0 km. The ADCIGs are flat over their respective angle apertures of −50° to +50°. The ADCIGs are filtered along the vertical direction using a Hamming window (0.006, 0.018, 0.040, 0.050 cycles/meter).

4.13 Stacked (a) PP and (b) PS images with incident angles from −50° to 50° with the vector-based prestack image condition. Direct waves were removed from the input common-source gathers before migration. Note the higher image resolution in (b) than in (a). Compare with Figure 4.9.
5.1 Acoustic or elastic wavefields represented as slices through a \( t-x-z \) volume. VSP data are represented with a \( t-z \) slice (yellow) at a constant \( x \) of the volume; an \( x-z \) wavefield snapshot (green) is an \( x-z \) slice at a constant \( t \). ....... 88

5.2 (a) is the input seismic trace (a Ricker wavelet) in the time domain; (b) is the Fourier transform of (a); (c) is the Hilbert transform of (a), notice that both (b) and (d) have the same symmetric amplitude spectra; only the phases are shifted by 90°. (e) is the complex trace constructed using equation 5.7 with the (a) (blue) as real part and (c) (red) as imaginary part, and (f) is the complex trace spectrum. The solid blue and dashed green curves in (b) (d) and (f) are the amplitude spectra and wrapped phases respectively. The complex trace (e) has a non-zero amplitude spectrum only for positive frequencies (f), and its amplitude is doubled compared to (b) and (d). The 0 frequency point is shifted to the middle of the spectra in Figure 5.2b, 5.2d and 5.2f for easier visualization. ....... 92

5.3 Two-layer P-wave velocity (\( v_p \)) S-wave velocity (\( v_s \)) and density (\( \rho \)) model for generating elastic VSP data (Figures 5.4a and 5.4b). The red spot is the source location; the green squares are every 14th receiver in a borehole. ....... 94

5.4 (a) and (b) are horizontal and vertical particle velocity components of the observed elastic VSP data; (c) and (d) are the horizontal and vertical components of the decomposed P-wave; (e) and (f) are the horizontal and vertical components of the decomposed S-wave; (g) and (h) are the down-going P-wave; (i) and (j) are the down-going S-wave; (k) and (l) are the up-going P-wave; (m) and (n) are the up-going S-wave. ....... 96

5.5 Flowchart of the procedures for the up/down wavefield separation. The input complex source or seismic traces can be generated using equation 5.7. For multi-component elastic wavefields, the forward and inverse Fourier transforms are performed on each component. ....... 97

5.6 (a) and (b) are snapshots of the horizontal and vertical components of 2D elastic wave propagation in a homogeneous elastic medium with a composite (P/S) source at the center; (c) and (d) are the components of the up-going P-waves; (e) and (f) are the components of the down-going P-waves; (g) and (h) are the components of the up-going S-waves; and (i) and (j) are the components of the down-going S-waves. ....... 99

5.7 Snapshots of horizontal and vertical particle velocities of elastic waves from two explosive and two rotational sources. The zoomed in part of the wavefront intersection part (in the green boxes) of the horizontal component with the extracted Poynting vectors, before and after P/S decomposition and up/down separation, are shown in Figure 5.8. ....... 102
5.8 (a) Part of the horizontal component of the elastic wavefield (in color) from Figure 5.7, superimposed with Poynting vectors before wavefield separations. After the P/S vector decomposition, followed by up/down direction separation, the wavefields and their corresponding Poynting vectors are decomposed into: (b) the up-going P-wavefield, (c) the down-going P-wavefield, (d) the up-going S-wavefield and (d) the down-going S-wavefield. 递给103

5.9 (a) P-wave velocity, (c) S-wave velocity and (e) density distributions used for isotropic elastic wavefield modeling; (b), (d) and (f) are smoothed (a), (c) and (e) respectively. This is a portion of the Marmousi2 model. 递给104

5.10 Elastic RTM images using crosscorrelation of corresponding components directly. (a) and (b) are \( I_{zz} \) and \( I_{xx} \) stacked images (left) and their corresponding representative ADCIGs (right) at four horizontal locations. 递给105

5.11 Elastic RTM images using only P/S decomposition. (a) and (b) are \( I_{pp} \) and \( I_{ps} \) stacked images (left) and their corresponding representative ADCIGs (right) at four horizontal locations. 递给107

5.12 Elastic RTM images using both P/S and up/down decompositions. (a) and (b) are \( I_{pp} \) and \( I_{ps} \) stacked images (left) and their corresponding representative ADCIGs (right) at four horizontal locations. 递给109

5.13 PP (a) and PS (b) images after Laplacian filtering of Figure 5.11a and 5.11b, respectively. 递给110

6.1 Sketches of rays in source (red) and receiver (blue) wavefields that generate images or artifacts in four model geometries. (a) and (b) are \( I_{dd} \) and \( I_{uu} \) images of a single-reflector model that both cause low-frequency noise. (c), (e) and (g) are correct \( (I_{du}, I_{du} \text{ and } I_{ud}) \) images respectively; (d), (f) and (h) are the corresponding mispositioned artifact \( (I_{ud}, I_{ud} \text{ and } I_{du}) \) images, respectively. 递给125

6.2 Two-reflector velocity model. The red dot is the source location; the blue triangles are every 14th receiver. 递给127

6.3 (a) is the common-source gather with the source at the center and top of the surface with the direct wave removed; (b) is the common-source gather in (a) after Hilbert transforming each trace in the \( t \) direction. (a) and (b) are the real and imaginary parts of the complex seismogram that is input to the RTM. Reflections labeled R1 and R2 are from the reflectors labeled R1 and R2 in Figure 6.2; reflection R3 is the first internal multiple between reflectors R1 and R2. 递给127

6.4 RTM images obtained using the acoustic data in Figure 6.3b and 6.3c and the true velocity model (Figure 6.2). (a) is the crosscorrelation image without up/down separation; (b) is the down-up image; the images labeled R1 and R2 correspond to the reflectors R1 and R2 in Figure 6.2. (c) is the up-down image; the amplitudes are much smaller than in (b), and note the artifact labeled R2'; (d) is the up-up image; (e) is the down-down image. (d) and (e) contain mainly low-frequency noise associated with the paths in Figure 6.1 (a and b). 递给129
6.5 Artifacts produced by internal multiples. Down-up (a) and up-down (c) images of Figures 6.4b and 6.4c with a higher amplitude plotting scale factor; (b) and (d) are their corresponding raypath sketches to illustrate the corresponding images/artifacts. The solid raypath m0 in (b) indicates how the internal multiples are generated; the dashed lines m1, m1’ and m2 are possible raypaths for imaging the multiples during RTM; M1 and M2 in (a) and (c) are the corresponding images/artifacts. M1’ is not labeled in (c) because it is imaged outside the model [see panel (d)].

6.6 P-wave velocity of the Sigsbee model. The red dots represent every 2nd source position. The receivers are evenly distributed from (0.0) km to (8.0, 0.0) km with spacing of 5 m.

6.7 Stacked RTM images of the Sigsbee model test with 20 sources using the true velocity model; (a) is the crosscorrelation image without up/down wavefield separation; (b) is the down-up image; (c) is the up-down image. Note the subsalt area (the red oval) is better imaged in (c) than in (b), because this area depends on mainly turning waves. However, for near-offset areas (e.g. the blue ovals) the up-down image (c) suffers more from the backscattering artifacts than (b).
LIST OF TABLES

2.1 Variables required for 2D vector decomposition algorithms and elastic wavefield extrapolation. ($M_{2u}$ is not included in the selective attenuation column as $M_{2u} = M_{1u}$). ................................. 28
CHAPTER 1

INTRODUCTION

1.1 Motivation and objectives

In multi-component seismic data, both P- and S-wave modes exist in each particle-velocity component, and separation of P- and S-waves is a necessary part of elastic migrations. A component-by-component cross-correlation imaging condition of unseparated source and receiver wavefields produces crosstalk artifacts between different wave modes during migration. Traditional separation methods with curl and divergence operators don’t preserve the wavefield vector component information. Thus the resultant migration images are no longer accurate. To keep both phase and amplitude accurate during PS separation, all vector components of the wavefield need to be preserved, which is called PS decomposition. However, the methodologies of PS decomposition are not fully analyzed and compared. Furthermore, there is no elastic image condition that directly uses the decomposed P and S vector wavefields to construct images.

The first objective of this dissertation is to analyze and compare various aspects of two PS vector decomposition algorithms. The second objective is to propose a new elastic reverse-time migration (RTM) image condition that uses the decomposed vector wavefields to construct PP and PS images and angle-domain common image gathers (ADCIGs). The third objective is to combine up/down separation with PS decomposition and further remove artifacts in the images. The fourth objective is to use up/down separation to analyze origins and positions of artifacts in acoustic RTMs.
1.2 Overview

Chapter 1 describes the motivation and the objectives (above), and an overview of each of the following chapters.

Chapter 2 describes two different methods for decomposing P- and S-waves in isotropic elastic wavefields. One is to apply strong attenuation to one (P or S) wave mode during propagation; the other is to use a decoupled isotropic elastic wave equation for wavefield extrapolation. The two methods are compared in terms of efficiency, accuracy and memory cost. We also demonstrate the procedure to perform P and S decomposition in $x$-$t$ gather data.

My contributions to Chapter 2 include initiating the idea of PS decomposition using attenuation, developing the codes, designing testing experiments, testing the algorithm, the initial manuscript writing and multiple rounds of revisions. The contributions of co-author G. A. McMechan include inspiring the main idea, discussions and analysis of the results, intensive editing and multiple revisions of the manuscript. The contributions of co-author Q. Zhang include improvement of the main ideas, and providing the computation environment at Repsol for some of the tests.

Chapter 3 describes the extension of a PS decomposition method from isotropic elastic wavefields to isotropic viscoelastic wavefields. The memory variables in the elastodynamic equations are rearranged to form decoupled forms to attenuate P- and S-waves separately. Together with the decoupled elastodynamic equation, the P- and S-waves in viscoelastic wavefields can be decomposed without losing accuracy.

My contributions to Chapter 3 include proposing the main idea, testing and developing the codes, the initial manuscript writing and multiple rounds of revisions. The contributions of co-author G. A. McMechan include improving the main idea, discussions and analysis of the results, intensive editing and multiple revisions of the manuscript.
Chapter 4 describes a new workflow of elastic RTM, which directly uses the decomposed P- and S-waves produced in Chapter 2, and applies a vector-based image condition to construct PP, PS images and ADCIGs.

My contributions in Chapter 4 include establishing the workflow for the vector-based elastic RTM, proposing the new image condition, writing the codes, testing the algorithms, the initial manuscript writing and multiple rounds of revisions. The contributions of co-author G. A. McMechan include improving the main idea, discussions and analysis of the results, intensive editing and multiple revisions of the manuscript.

Chapter 5 combines the up/down separation with PS decomposition, and decomposes the multi-component vertical seismic profile (VSP) data and propagating wavefields into P and S parts, and further to up-going and down-going parts, respectively. This technique helps to avoid low-frequency noise in elastic RTMs. Poynting vectors also demonstrate better stability after PS and up/down decompositions, especially where wavefronts overlap.

My contributions in Chapter 5 include initiating the main idea, writing the codes, testing the algorithms, the initial manuscript writing and multiple rounds of revisions. The contributions of co-author G. A. McMechan include improving the main idea, discussions and analysis of the results, intensive editing and multiple revisions of the manuscript. The contributions of co-author F. Xie include inspiring the idea of using complex traces for up/down separation, and discussion of the results.

Chapter 6 describes the applications of up/down separation in acoustic RTM artifacts analysis. We decompose the acoustic RTM image into four (down-up, up-down, up-up and down-down) images, and find that artifacts may exist in all the decomposed images, but their positions and types are different. We explain and illustrate the causes of those artifacts with sketches and numerical tests, and propose to use decomposed images for target interpretations.

My contributions in Chapter 6 include initiating the main idea, writing the codes, testing the algorithms, the initial manuscript writing and multiple rounds of revisions. The
contribution of co-author G. A. McMechan include improving the main idea, discussions and analysis of the results, intensive editing and multiple revisions of the manuscript. The contributions of co-author F. Xie include discussion of the results.

Chapter 7 concludes the dissertation with a summary of the results of the previous chapters, and suggests future work.

1.3 Publication status

All papers have been published, and the publication information is provided below:

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1.4 Declaration

If this manuscript includes authentic copies of papers already published or submitted, these are identified by an asterisk at the title of the relevant chapter. Connecting text which provides a logical bridge between different manuscripts has been added. The introductory material to the published papers (above) describes my contribution to the work (and acknowledges the contribution of other coauthors). The signatures of the Supervising Committee which precede all other material in this manuscript attest to accuracy of this statement.
CHAPTER 2
COMPARISON OF TWO ALGORITHMS FOR ISOTROPIC ELASTIC P AND S VECTOR DECOMPOSITION∗

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2.1 Abstract

P and S wavefield-separation is necessary to extract PP and PS images from prestack elastic reverse-time migrations. Unlike the traditional separation methods that use curl and divergence operators, which do not preserve the wavefield vector component information, we do P and S vector decomposition, which preserves the same vector components that exist in the input elastic wavefield. The amplitude and phase information is automatically preserved, so no amplitude or phase corrections are required. We consider two methods to realize P and S vector decomposition: selective attenuation and decoupled propagation. Selective attenuation uses viscoelastic extrapolation, in which the Q values are used as processing parameters, to remove either the P-waves or the S-waves. Decoupled propagation rewrites the stress and particle velocity formulation of the elastic equations into separate P- and S-wave components. In both methods, the decomposition is realized during the extrapolation of an elastic wavefield. These algorithms can also perform P and S decomposition in x-t gather data by extrapolating the data downward from the receivers, during which the decomposition is performed, and then back upward to record the decomposed P- and S-waves at the receivers. Comparisons of the two methods in terms of efficiency, accuracy and memory show that both can separate P- and S-waves in the vector-domain. The decoupled propagation is preferable in terms of speed and memory cost, but is applicable only to elastic propagation.

2.2 Introduction

Separation of P- and S-waves is a necessary part of elastic migrations. In multi-component seismic data, both P and S wave modes exist in each particle-velocity component, so using a component-by-component crosscorrelation imaging condition of unseparated source and receiver wavefields will lead to crosstalk artifacts between different wave modes during migration.
Many methods have been proposed to solve the separation problem. Helmholtz decomposition is widely used to separate the elastic wavefield (Morse and Feshbach, 1953; Clayton, 1981; Mora, 1987). Sun (1999) and Sun and McMechan (2001) performed elastic extrapolation and used divergence and curl operators to separate the P and S wavefields near the surface, followed by two acoustic reverse-time migrations (RTMs), one for the P-P reflection and one for the converted P-S reflection. Dellinger and Etgen (1990) propose divergence-like and curl-like operators in the wavenumber domain based on Helmholtz theory, and the Christoffel equation to realize the decomposition in anisotropic homogeneous elastic media. Yan and Sava (2008a) transform the separation operator from the wavenumber domain to the space domain, and formulate a filter to separate wavefields in heterogeneous transversely isotropic media with a vertical symmetry axis.

All the above separation schemes involve using divergence and curl, or similar, operators on the propagating elastic wavefields, which generate a scalar wavefield (divergence) and a vector wavefield (curl) with different physical meaning from the input elastic wavefield. Thus the amplitude and phase are changed, and much effort needs to be spent on corrections to obtain well-focused, true-amplitude images from prestack elastic RTM. To avoid this problem, Zhang and McMechan (2010) extend Dellinger and Etgen (1990) theory to separate wavefields in VTI media into P- and S-particle components (vector decomposition), which is desirable, but computationally expensive because of the forward and inverse fast Fourier transforms (FFTs) at each time step. Cheng and Fomel (2014) use a low-rank approximation to reduce the number of FFT operations per time step. However, for practical applications in isotropic media, we seek simpler and less expensive P and S decomposition algorithms.

In this paper, two affordable yet accurate, algorithms that do not use FFTs are analyzed and compared; both methods decompose the vector P and S waves in an isotropic elastic wavefield during wavefield extrapolation. The first algorithm is selective attenuation, in which attenuation is used as a tool to decompose the wavefields; the viscoelastic extrapolation
is implemented with memory variables (Carcione et al., 1988a). The P and S decomposition is achieved during the extrapolation by applying strong attenuation to either P- or S-waves while preserving the other wave mode.

The second method is decoupled propagation. Ma and Zhu (2003) solve the decoupled isotropic elastic wave equations during extrapolation in a pseudospectral formulation. Zhang et al. (2007) use a decoupled 2D staggered-grid stress and particle-velocity formulation to decompose the elastic wavefield. Xiao and Leaney (2010) use different, but equivalent, equations to do wavefield-decomposition for elastic interferometric RTM of VSP data. Both decomposition methods are embedded in wavefield extrapolation, and the output P- and S-wavefields are the same vector components as the input wavefields. The advantages of vector-domain elastic wavefield-decomposition suggest direct applications in elastic prestack RTM, and the relation between the decomposed elastic wavefield vectors and propagation directions can be applied in generating PP and PS angle-domain common-image gathers (ADCIGs). However, these are beyond the scope of this paper, and will be discussed elsewhere.

This paper is organized as follows. First, the methodologies of selective attenuation and decoupled propagation methods are introduced, followed by tests on synthetic data to show the decomposition results. Then we present a systematic comparison of the two methods in terms of accuracy, speed, memory usage and numerical stability criteria. Finally, inspired by the work of Sun (1999), we decompose $x$-$t$ multicomponent data into P- and S-seismograms using decoupled propagation.

### 2.3 Methodology

To demonstrate the necessity to do wavefield-decomposition, we use divergence ($\nabla \cdot$) and curl ($\nabla \times$) operators (Dellinger and Etgen, 1990) as the benchmark method to separate the wavefield.
\[ \theta = \nabla \cdot \mathbf{u}, \]  
\[ \varphi = \nabla \times \mathbf{u}, \]

where \( \mathbf{u} \) is the displacement of the elastic wavefield, and \( \theta \) and \( \varphi \) represent the separated P- and S-waves (Aki and Richards, 1980). Particle velocity can be substituted for displacement. Both \( \theta \) and \( \varphi \) are dimensionless and have no physical interpretation. However simple and elegant this method is, the divergence and curl operators don’t preserve the features of the input elastic wavefield. In equation 2.1, \( \theta \) is a scalar, and in equation 2.2, \( \varphi \) is a vector that is locally perpendicular to the S-wave polarization. The spatial derivatives in \( \nabla \cdot \) and \( \nabla \times \) produce a 90° phase shift in the wavelet, thus making both the amplitude and phase produced by any imaging condition unphysical, and migrations difficult to implement and interpret, especially for P-S converted reflections. To overcome this problem, and to obtain accurate migration images, corrections to both amplitude and phase need to be made. A Hilbert transform can be used to correct the phase information (Sun et al., 2001), and amplitudes need to be calibrated with knowledge of P- and S-wave velocities (Sun et al., 2011; Nguyen and McMechan, 2015) using

\[ \frac{|u_p|}{|u_s|} = \frac{\theta}{v_s} \frac{v_p}{v_s} \]

where \( |u_p| \) and \( |u_s| \) are the magnitudes of the displacement vectors of the P- and S-waves; \( v_p \) and \( v_s \) are P- and S-wave propagation velocities, respectively.

The corrections for divergence and curl involve additional computation, and may introduce artifacts. A better solution is to decompose the elastic wavefield while preserving the original particle components that are present in the input elastic data, so both amplitude and phase are naturally accurate. In the following two subsections, two methods for wavefield-decomposition are introduced, investigated and illustrated.
2.3.1 Selective Attenuation

The basic idea of selective attenuation is to modify the viscoelastic extrapolator to attenuate one wave mode during the wavefield extrapolation, while preserving the other. Viscoelastic extrapolations have previously been used to simulate the anelastic phenomena of the real earth. The theory of linear viscoelasticity is based on the superposition of relaxation mechanisms (Liu et al., 1976; Emmerich and Korn, 1987). The standard linear solid model implements the process of attenuation (Zener, 1948). In this formulation, the moduli become functions of time and frequency.

\[ \sigma = \Psi \star \dot{\epsilon} \]  

(2.4)

where \( \sigma \) is the stress tensor, \( \epsilon \) is the strain tensor, \( \Psi \) is the modulus, and the dot denotes a time derivative. \( \sigma, \epsilon \) and \( \Psi \) are all time-dependent. Dissipation of energy occurs when stress and displacement are out of phase (e.g., Guéguen and Palciauskas, 1994). Carcione et al. (1988a) replace the convolution operation in equation 2.4 by using memory variables. Memory variables allow the computation of synthetic wavefields for models with arbitrary spatial distributions of quality factors. Xu and McMechan (1995) use composite memory variables to reduce the number of memory variables to be stored.

The quality factor \( Q \) characterizes the attenuation of waves in earth materials, and is a measure of the number of wavelengths through which a wave must propagate in a material for its amplitude to decrease by \( 1/e \) (e.g., Robertsson et al., 1994). Earth materials have been shown to have nearly constant quality factors (\( Q \) values), over the exploration seismic frequency range (McDonal et al., 1958; Bourbie et al., 1987). Usually, more than one relaxation mechanisms are combined to construct a desired \( Q \) behavior as a function of frequency. However, this is not necessary for the purpose of wavefield-decomposition, in which \( Q \) is treated as a processing parameter; thus, having only one relaxation mechanism
is sufficient, and the computations are simplified. Figure 2.1 shows $Q^{-1}$ (for P-waves) as a function of frequency with only one relaxation mechanism, and the frequency spectrum a representative source. The $Q^{-1}$ curve has one apex, and we can set its position to coincide with the dominant frequency of the source. Thus, the part of the source that has the most energy also has the strongest attenuation; other frequencies correspond to weaker attenuations. The apex value of the $Q^{-1}$ curve (at $Q^{-1} \approx 0.5$) in Figure 2.1 is sufficient to attenuate the P-wave quickly. Field data often has a nearly flat spectrum in the seismic frequency range, and using more relaxation mechanisms for attenuation is an option to better fit the data frequency spectrum; the computational cost will be increased correspondingly. If the shape of the spectral function used to attenuate the P- or S-waves does not match that of the data, it will take more time steps to achieve decomposition, but it will still work. Decreasing Q also increases the phase velocity at high frequencies (Carcione et al., 1988a), which may cause numerical instability problems when using finite-difference schemes for wavefield extrapolation; the computational grid increment and time step can be adjusted to ensure stability.

There are different methods for parameterization of viscoelastic moduli. Carcione et al. (1988a,b); Carcione (1993) use relaxed (and unrelaxed) bulk and shear moduli to describe the model; Robertsson et al. (1994) use P- and S-wave moduli. Because of the differences in parameterizations, the details of the calculations for the memory variables are also different. However, for the purpose of P and S decomposition, we choose to parameterize the model with P and S-wave moduli, in which case, the memory variables can be grouped into P- and S-parts. The algorithm of selective attenuation can be deduced from the viscoelastic constitutive equations, for an isotropic 2D medium (Robertsson et al., 1994), that relate stresses and particle velocities. Thus,

$$\frac{\partial \sigma_{xx}}{\partial t} = M_{1u} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) - 2M_{2u} \frac{\partial v_z}{\partial z} + \epsilon_{11},$$  \hspace{1cm} (2.5)
Figure 2.1. P-wave $Q^{-1}$ as a function of frequency with one relaxation mechanism (the solid line) and the frequency spectrum of the source (the dashed line).

\[
\frac{\partial \sigma_{zz}}{\partial t} = M_{1u} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) - 2M_{2u} \frac{\partial v_x}{\partial x} + e_{22},
\]

(2.6)

and

\[
\frac{\partial \sigma_{xx}}{\partial t} = M_{2u} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) + e_{12}.
\]

(2.7)

The memory variables, $e_{ij}$ in equations 2.5-2.7, are obtained recursively over time steps from

\[
\frac{\partial e_{11}}{\partial t} = \frac{1}{\tau_{\sigma}} (M_{1r} - M_{1u}) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) - \frac{2}{\tau_{\sigma}} (M_{2r} - M_{2u}) \frac{\partial v_z}{\partial z} - \frac{e_{11}}{\tau_{\sigma}},
\]

(2.8)

\[
\frac{\partial e_{22}}{\partial t} = \frac{1}{\tau_{\sigma}} (M_{1r} - M_{1u}) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) - \frac{2}{\tau_{\sigma}} (M_{2r} - M_{2u}) \frac{\partial v_x}{\partial x} - \frac{e_{22}}{\tau_{\sigma}},
\]

(2.9)

and

\[
\frac{\partial e_{12}}{\partial t} = \frac{1}{\tau_{\sigma}} (M_{2r} - M_{2u}) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) - \frac{e_{12}}{\tau_{\sigma}}.
\]

(2.10)

Here, $M_{1r}$ and $M_{2r}$ are relaxed (elastic) moduli for P- and S-waves (indicated by subscript 1 and 2, respectively), $M_{1u}$ and $M_{2u}$ are the corresponding unrelaxed (viscoelastic) moduli, $\sigma_{ij}$ is stress, $v_i$ is the $i$th component of the particle-velocity vector, and $\tau_{\sigma}$ is the stress relaxation time.
The above formulation simulates attenuation of both P- and S-waves, but to realize the P and S wavefield-decomposition, we need to preserve one wave mode during extrapolation. We manipulate the formulation to make the energy dissipation occur only for one wave mode by making $M_{1r} = M_{1u}$ or $M_{2r} = M_{2u}$, and thereby decomposing the P and S waves. The viscoelastic formulation leads to dispersion as well as attenuation; however, the associated dispersion can be disregarded as long as it doesn’t cause a numerical instability problem, because it occurs only in the wave mode that is being attenuated.

To realize the P and S vector decomposition, we can choose to attenuate either P- or S-waves, but we find that P-wave attenuation is more efficient because, if all the S-wave moduli are elastic ($M_{2r} = M_{2u}$), then only one memory variable is needed (in both 2D and 3D), which is much faster and more efficient than solving the original viscoelastic wave equations. In contrast, attenuating the S-waves requires that three memory variables be calculated in 2D, and six in 3D. As a result, we use only P-wave attenuation to represent the selective attenuation method in the following tests and comparisons. To attenuate the P-waves, we define a new memory variable, $e_{PP}$, at each grid point, by

$$\frac{\partial e_{PP}}{\partial t} = \frac{1}{\tau_\sigma}(M_{1r} - M_{1u})(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}) - \frac{e_{PP}}{\tau_\sigma}; \quad (2.11)$$

thus, the 2D memory variables can be simplified to

$$e_{11} = e_{22} = e_{PP}, \quad (2.12)$$

and

$$e_{12} = 0. \quad (2.13)$$

With strong attenuation of the P-waves, after a few time steps, the extrapolated wavefields contain only S-waves in all particle-velocity and stress components. To get the decomposed P-waves, a complete elastic extrapolation needs to be performed concurrently, and the particle-velocity components of the P-waves are obtained by subtraction of the preserved S wavefield from the complete propagating elastic particle-velocity wavefield at the same time step.
2.3.2 Decoupled Propagation

Decoupled propagation is based on the assumption, implementation and application of a decoupled isotropic elastic extrapolation. Ma and Zhu (2003) applied the elastic numerical solution for decoupled P- and S-waves in a pseudospectral solution. Zhang et al. (2007) and Xiao and Leaney (2010) use 2D staggered-grid, stress-particle velocity formulations with a similar idea to decompose the elastic wavefield. Following Xiao and Leaney (2010), the stress-particle velocity formulation of the elastodynamic equations for decoupled isotropic elastic propagation is

\[
\frac{\partial \sigma_{xx}}{\partial t} = [(\lambda + 2\mu)(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z})] - 2\mu \frac{\partial v_z}{\partial z}, \quad (2.14)
\]

\[
\frac{\partial \sigma_{zz}}{\partial t} = [(\lambda + 2\mu)(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z})] - 2\mu \frac{\partial v_x}{\partial x}, \quad (2.15)
\]

\[
\frac{\partial \sigma_{zx}}{\partial t} = \mu \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right), \quad (2.16)
\]

\[
\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z} \right), \quad (2.17)
\]

and

\[
\frac{\partial v_z}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right), \quad (2.18)
\]

where \(\sigma_{ij}\) is the stress, \(v_i\) is particle-velocity, and \(\lambda\) and \(\mu\) are Lamé parameters.

The time derivative of the normal stress (in the square brackets in equations 2.14 and 2.15) corresponds to the P-wave stress, so it can be calculated separately and given the new notation \(\sigma_P\), to replace \(\sigma_{xx}\) and \(\sigma_{zz}\) where

\[
\frac{\partial \sigma_P}{\partial t} = (\lambda + 2\mu)(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}). \quad (2.19)
\]

The P-wave particle-velocity components \(v_{xP}\) and \(v_{zP}\) are calculated from \(\sigma_P\) from equations 2.17 and 2.18 by finite differencing to give

\[
\frac{\partial v_{xP}}{\partial t} = \frac{1}{\rho \partial x} \frac{\partial \sigma_p}{\partial x}, \quad (2.20)
\]
and

$$\frac{\partial v_{zP}}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma_p}{\partial z}. \quad (2.21)$$

This gives a complete decomposition of the P-wavefield and, similar to the selective attenuation method, we can get the S-wavefield by subtracting the P-wavefield from the complete wavefield component-by-component, so

$$v_{xS} = v_x - v_{xP}, \quad (2.22)$$

and

$$v_{zS} = v_z - v_{zP}. \quad (2.23)$$

The decoupled propagation obtains the P-waves by utilizing the divergence operator that already exists within the elastodynamic wave equations 2.14 and 2.15, via equations 2.19, 2.20 and 2.21, so the decomposition process is a part of the wavefield propagation process, and both amplitude and phase are accurately preserved. Zhang et al. (2007) use a different form of the decoupled equations, which can be proven to be equivalent to equations 2.19-2.23 (see the Appendix in section 2.10).

### 2.4 Test with synthetic Data

In this section, both selective attenuation and decoupled propagation are tested with data from two different elastic models, together with a test on data from a homogeneous elastic model using divergence and curl operators to demonstrate the advantages of vector decomposition. For all the following tests, we use a 2D staggered-grid finite-difference solution (Virieux, 1986) to solve the stress-particle velocity formulation of the elastodynamic equations. The advantage is that it does not use derivatives of the elastic moduli, and thus is more efficient and accurate than the traditional non-staggered elastic wave equation. We use eighth-order finite-difference equations to calculate spatial derivatives at each grid
point within the model including the edges; there will not be any significant edge reflections, because convolutional perfectly matched layer (CPML) absorbing boundary conditions (Komatitsch and Martin, 2007) with a width of 20 grid points inside the model are used on all four grid edges. To have enough grid points for eighth-order finite differencing of the derivatives in the elastodynamic equations, the virtual top of the model is defined at the 5th grid point at depth \( z \) below the inner bound of the top absorbing layer.

The first 2D test is performed on a \( 256 \times 256 \) homogeneous isotropic elastic model with grid increments \( h = 5 \) m and time increments \( dt = 0.5 \) ms. A composite source, which generates both P- and S-waves simultaneously, is placed at the center of the grid. Figure 2.2a shows snapshots of the resulting elastic wavefield at five different times. The P- and S-waves are propagating at different velocities and have different polarizations in their horizontal and vertical components.

We first separate the P and S-waves in the 2D, 2-component elastic wavefield (Figure 2.2a) using divergence and curl operators (Figure 2.2b). The P and S separation is good, but the original 2-component (\( x \) and \( z \)) elastic wavefield becomes one component for P and one for S. The P-wave becomes a scalar wavefield, and the S-wave become a vector wavefield with only one non-zero component; the original vector component information is lost. Without the amplitude correction using equation 2.3, and the 90° phase correction, both the phase and amplitude information are changed from those in the input data.

2.4.1 Decomposition by selective attenuation

Next we change the medium from elastic to viscoelastic with strong attenuation of P-waves using \( Q^{-1} \approx 0.5 \) as shown in Figure 2.1. Since the P-waves are attenuated quickly, the snapshots in Figure 2.3b are the decomposed S-wavefield. The subtraction of Figure 2.3b from Figure 2.2a gives the decomposed vector P-wavefield snapshots in Figure 2.3a.
Figure 2.2. (a) Snapshots of 2D wave propagation in an elastic homogeneous medium with a composite P and S source at the center. Vertical and horizontal components of the wavefields are recorded. (b) Separation results with the divergence (upper panels) and the curl (lower panels) operators. Note the large differences in the relative amplitudes between (a) and (b), because they are not the same physical waves. Amplitudes along the dashed lines here, and in Figures 2.3 and 2.4 are shown in Figure 2.5.
Figure 2.3. Snapshots of wave propagation in a viscoelastic medium with strong P-wave attenuation. (a) Decomposed P-waves; (b) decomposed S-waves. Compare with Figures 2.2 and 2.4.
2.4.2 Decomposition by decoupled propagation

Figure 2.4 shows the decomposed P- and S-waves using decoupled propagation. Compare with the snapshots calculated by selective attenuation in Figure 2.3; even though they use different methods, both yield very similar, clean and accurate decomposition results for both P- and S-waves. The main difference is that some small residuals remain in the first snapshots of selective attenuation, because attenuation (no matter how strong) cannot be achieved instantly. In both Figures 2.3 and 2.4, both vertical and horizontal components of the elastic wavefields are preserved in the decomposition, and there are no phase or amplitude changes, so no corrections are needed.

2.5 Evaluation of algorithm performance

Both selective attenuation and decoupled propagation decompose the wavefield into their respective P- and S-vector components. It will be valuable to know which is more suitable for application; the comparisons below are performed to evaluate accuracy, speed, memory requirements, and stability.

2.5.1 Accuracy

We consider accuracy to be the most important basis for evaluation of the decomposition methods. To provide a criterion to test the accuracy of the decomposition results, we return to the simulation in Figure 2.2a, but the source is changed to an explosive source to generate pure P-wave snapshots, and then is changed to a shear source to get pure S-wave snapshots at the same time steps as those shown in Figure 2.2a. Because the medium is homogeneous, no conversions occur during the elastic wavefield extrapolation. If the decomposition is accurate, the results should be the same as those generated in the pure P- or S-wave snapshots, and the residual can be used as a quantitative measure of accuracy.
Figure 2.4. Decomposed snapshots of wave propagation using decoupled propagation. (a) Decomposed P-wavefield; (b) decomposed S-wavefield. Compare with Figures 2.2 and 2.3.
We define the wavefield residual to be the sum of the absolute value over all grid points and components, of the difference between the pure-wave-mode P and S particle-velocity snapshots and the corresponding decomposed P and S snapshots. In Figure 2.5 the pure P- or S-waves (the dashed red lines) are plotted along with the decomposition results (the solid blue lines), for both decomposition methods, extracted along the dashed line trajectories in Figures 2.2a, 2.3 and 2.4. The good match between the pure and the decomposed waves indicates both the absolute accuracy of the decomposed waves, and the equivalent accuracy for both decomposition algorithms; the overlain blue and red lines are visually indistinguishable.

Figure 2.6 shows the particle-velocity residuals in the homogeneous medium; the residuals in inhomogeneous media are similar. Since the attenuation method can not decompose the wavefields instantly, compared with decoupled propagation, the selective attenuation needs more time steps to complete the vector decomposition. The decoupled propagation has near-zero residuals from the beginning.

Both vector decompositions are effective in an inhomogeneous velocity model provided that the model is sufficiently smoothed to avoid generation of reflections, converted waves, and head waves during the extrapolations. The presence of these secondary waves is particularly undesirable in the context of subsequent application reverse-time migration, where they produce artifacts. To illustrate, consider the simple elastic model in Figure 2.7a, which has a single flat reflector at 2.6 km depth. A rotational source, located at (x,z) = (2.0, 2.0) km generates only S-waves which, as they hit the reflector, produce S-to-P converted waves and evanescent (head) waves as well as reflected waves (Figure 2.8a). Applying the decomposition algorithms to these (forward propagating) waves produces the snapshots at 0.5 s shown in Figure 2.8a. The dashed ovals indicate positions of S-to-P wave conversions and S-wave reflections at the reflector. Figure 2.8b shows the same information, obtained using the smoothed velocity model in Figure 2.7b; the artifacts produced by both decomposition algorithms are smaller in the smoothed model.
Figure 2.5. Comparison of waveforms along the dashed lines in Figure 2.2a, 2.3 and 2.4 at 0.5 s. (a) and (b) are horizontal and vertical components of the elastic wavefield (without vector decomposition) from Figure 2.2a. (c) and (d) contain the P-waves, and (e) and (f) contain the S-waves, both decomposed using selective attenuation (the solid blue lines) and the pure P and S waves (the dashed red lines), respectively. (g) and (h) contain the P-waves, and (i) and (j) contain the S-waves, both decomposed using decoupled propagation (the solid blue lines) and the pure P and S waves (the dashed red lines), respectively.
Figure 2.6. The dotted line shows the residuals using the selective attenuation (of P-waves). The solid line shows the residuals using decoupled propagation.

Figure 2.7. (a) True and (b) smoothed velocity model for elastic wavefield extrapolation and P and S decomposition. The red dots are rotational sources. In (a) the upper layer has P- and S-velocity of 1.70 km/s and 1.20 km/s, and the lower layer has P- and S-velocity of 1.90 km/s and 1.40 km/s.
Figure 2.8. Snapshots at 0.5s of elastic wavefield and P and S decomposition results in a model with one horizontal reflector at z = 2.6 km (a), and the corresponding smoothed model (b). The models are shown in Figure 2.7. In both (a) and (b), the leftmost pair contains the vertical and horizontal components of the elastic wavefield (without decomposition), and the next two pairs, contain the decomposed P- and S-waves using selective attenuation. The two rightmost pairs, contain the decomposed P- and S-waves using decoupled propagation. The sources for both decompositions are rotational, and generate only S-waves. Conversions to P-waves occur at the reflector if using the true velocity model. Both selective attenuation and decoupled propagation decompositions produce artifacts (in the dashed ovals) at the reflector in (a), because of the coupling of P- and S-waves and the generation of head waves, but these artifacts are much reduced when using the smoothed velocity model, in (b).
2.5.2 Speed

Time efficiency is a key consideration for commercial applications. Both the selective attenuation and decoupled propagation vector decomposition methods are embedded in the extrapolations, which means both are more costly than solving only the elastic wavefield extrapolation. We evaluate the relative time efficiency of the attenuation and decoupled algorithms by comparing their computation times (including decomposition), with the time for elastic extrapolation alone (without decomposition). The CPU used is a single core from an AMD Phenom™ 9850 Quad-Core 2.6 GHz Processor. Figure 2.9 shows the time consumed by each method for three different grid sizes. The decoupled propagation has better time efficiency than selective attenuation by approximately a factor of 2, mainly because selective attenuation needs a second (elastic) extrapolation if both decomposed P- and S-waves are needed.

2.5.3 Memory Requirement

The memory occupied by elastic wavefield extrapolation with vector decomposition is larger than without decomposition (Table 2.1). If the output wavefields are the decomposed particle-velocity vectors, then we need to store at least one additional two-component elastic wavefield per time step: the decomposed P- or S-vector wavefield (the other wave mode
can be obtained by subtraction) no matter what vector decomposition method we choose. Additional memory might be necessary depending on the algorithms. The P-wave stress grid $\sigma_P$, as used in decoupled propagation, also needs to be saved during wavefield extrapolation. For selective attenuation (only P-wave attenuation is considered in this section), one value of the memory variable $e_{P P}$ needs to be stored for each grid point. To obtain both decomposed P- and S-wave modes, a complete elastic extrapolation needs to be carried out simultaneously with the visco-elastic extrapolation for subtraction at each time step; thus, the numbers of particle velocities and stresses are doubled, and we use a suffix $q$, after the viscoelastic stresses, to distinguish from the elastic ones in Table 2.1.

Because Q is only a processing parameter, a fixed relaxation time is used for all grid points to save RAM storage. The total size of memory needed is proportional to the number of wavefields and the size of each wavefield. Table 2.1 shows the number of variables that need to be kept in RAM by each wavefield-decomposition algorithm; in 2D, the selective attenuation needs 27% more RAM than decoupled propagation.

Since wavefield extrapolation with selective attenuation implemented gives only one wave mode (P or S) depending on which is attenuated, and we use subtraction to get the other wave mode, the complete elastic wavefield without decomposition also needs to be computed. However, if only one wave mode is needed for later processing, then the elastic wavefield without decomposition is no longer required, and the computational cost is correspondingly reduced.

2.5.4 Numerical Stability

The stability criterion influences how we choose the extrapolation parameters. Although both selective attenuation and decoupled propagation are embedded in extrapolation, they have different stability criteria as they involve finite-difference solutions of different equations. Because of the velocity dispersion in the viscoelastic scheme used in the selective
Table 2.1. Variables required for 2D vector decomposition algorithms and elastic wavefield extrapolation. \((M_{2u} \text{ is not included in the selective attenuation column as } M_{2u} = M_{1u}).\)

<table>
<thead>
<tr>
<th>Methods</th>
<th>Elastic Extrapolation</th>
<th>Decoupled Propagation</th>
<th>Selective Attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td>(\lambda, \mu, \rho)</td>
<td>(\sigma_{xx}, \sigma_{zz}, \sigma_{zx})</td>
<td>(M_{1r}, M_{2r}, M_{1u}, \rho)</td>
</tr>
<tr>
<td></td>
<td>(v_x, v_z)</td>
<td>(v_{px}, v_{pz}, \sigma_p)</td>
<td>(\sigma_{xx}, \sigma_{zz}, \sigma_{zx})</td>
</tr>
<tr>
<td>Total number</td>
<td>8</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

attenuation algorithm, the stability criterion has to be computed using the highest phase velocity \(v_{\text{max}} = \sqrt{\frac{M_{1u}}{\rho}}\), which is obtained at infinite frequency (Robertsson et al., 1994; Dablain, 1986; Levander, 1988); so the stability depends on the amount of attenuation that is applied for decomposition. Figure 2.10 shows the ratio, of P-wave velocities in a homogeneous viscoelastic medium of velocity \(V(f)\) (which is a function of frequency, \(f\)), to the constant velocity \((V)\) in the corresponding elastic medium. For the attenuation curve in Figure 2.1, the maximum viscoelastic P-wave velocity becomes \(\sim 1.6\) times the maximum elastic P-wave velocity, thus making it more expensive to satisfy the stability criterion. As shown in Figure 2.1, using low \(Q\) would make the attenuation of the unwanted waves more efficient, but would require a finer grid increment to insure stability. So the optimal combination is model dependent.

2.5.5 Comparison using Marmousi2 data

To demonstrate that both selective attenuation and decoupled propagation work in inhomogeneous as well as homogeneous media, a more salient test is performed on data from a part of the 2D elastic Marmousi2 model (Figure 2.11). The original model is redefined with grid increments \(h = 5\) m. The composite (P and S) source is placed at \(z = 0.3\) km and \(x = 1.6\) km; the water layer at \(z < 0.3\) km is replaced with an absorbing zone and is not shown
Figure 2.10. The ratio of wave velocities as a function of frequency in a homogeneous viscoelastic medium \([V(f)]\) and in the elastic medium \((V)\) with the same relaxed (elastic) modulus \(M_{1r}\).

in the snapshots. The time increment in the extrapolation is \(dt = 0.5\) ms. Figure 2.12a shows a snapshot of the elastic particle-velocity wavefield components before decomposition. The parameter models \((\lambda, \mu\) and density) need to be smoothed to do the decomposition so there are no extra reflected or converted waves produced during extrapolation (which is also a prerequisite in RTM to reduce migration artifacts). Figure 2.12b and 2.12c are the decomposed P- and S-wavefields using P-wave attenuation. Figure 2.12d and 2.12e are the decomposed P- and S-wavefields using decoupled propagation. Both selective attenuation and decoupled propagation effectively decompose the P- and S-waves into their vertical and horizontal components. Both methods would work similarly well in 3D.
2.6 P and S decomposition in $x$-$t$ gather data

To generate decomposed P- and S-wavefields from recorded 2C $x$-$t$ domain data, an additional step can be performed after P and S decomposition of a receiver wavefield in the space domain, as described by Sun and McMechan (2001). The P- and S-wavefields can be saved during the downward extrapolation in the space domain at a line of virtual receivers, and then input into upward extrapolations of the P- and S-wavefields, separately, to reconstruct a P-wave vector and an S-wave vector in the $(x, z, t)$ data domain. Sun and McMechan (2001) implementation used divergence and curl, but we can now do the decomposition with either of the vector algorithms, so that amplitude and phase of the vector components are both decomposed and preserved.

We use a homogeneous model for the extrapolations so there will be no mode conversions, and the model needs to be solid to propagate S-waves as well as P-waves. Any effects of the downward extrapolation in the homogeneous model will be undone in the subsequent upward extrapolation, so the decomposition results are not sensitive to the elastic model used for
Figure 2.12. Snapshots of the wave propagation in a portion of the elastic Marmousi2 model in Figure 2.11 at time = 0.75 s, using the true P-wave velocity model as the background with wavefield snapshots overlapped with 50% transparency. All snapshots are particle-velocity components; the left column is $v_x$ and the right column is $v_z$. (a) is the complete elastic particle-velocity wavefield without P and S decomposition; (b) is the decomposed P-wavefield using selective attenuation; (c) is the decomposed S-wavefield using selective attenuation; (d) is the decomposed P-wavefield using decoupled propagation; and (e) is the decomposed S-wavefield using decoupled propagation.
the extrapolations. We can choose either selective attenuation or decoupled propagation for the decomposition algorithm. As the decoupled propagation has better efficiency and doesn’t have a time requirement to complete decomposition (Figure 2.6), we apply only the decoupled propagation to do the decomposition for the $x$-$t$ data in this example. We use the two-component particle-velocity as the input seismogram for both downward and upward extrapolations. The stress components are not required for boundary conditions as long as their values are sufficiently small at the last time step (Nguyen and McMechan, 2015); after a few time steps of reconstruction, the stress components, computed from the particle velocities, will be consistent with the particle velocities via equations 2.14-2.21. The procedure is as follows:

1) Downward extrapolation of a source gather (a 2-component particle-velocity seismogram) into a homogeneous solid medium. Decoupled elastic wave equations 2.14-2.21 are used to obtain stresses and particle velocities iteratively for elastic wavefield extrapolation; the calculation of the S-wave particle velocities in equations 2.22 and 2.23 can be omitted because the S-wave seismogram can be obtained by subtraction in step 3 (below) after the P-wave components are decomposed.

2) Save the P-wave particle-velocities as time-slice seismograms, at a line of virtual subsurface receivers below the true receivers, to be used in the P-S decomposition. The depth of the virtual receivers $z \geq 4h$ (where $h$ is the grid increment); another 4 grid points needed for eighth-order finite differencing in the $z$-direction lie above the true receiver line. Four grid points are enough for decomposition using decoupled propagation, since with this method, the decomposition occurs instantaneously; more are needed when using selective attenuation, depending on the amount of attenuation being applied.

3) Use the P-wave particle-velocity seismogram saved at the virtual receivers for use in boundary conditions to upward extrapolate the wavefield; the original elastic wavefield extrapolator (equations 2.14-2.18) can be used, because the medium is homogeneous,
the upward extrapolated wavefield contains only P-waves, and no conversions will occur. Then, the P-wave particle-velocity seismogram is recorded at the same time steps and receiver positions as the original seismogram, and subtraction of the P-wave seismogram from the original seismogram yields the decomposed S-wave particle-velocity seismogram.

As an example, we use decoupled propagation to demonstrate the P and S vector decomposition of $x$-$t$ data for a part of the Marmousi2 model (Figure 2.11). An explosive source with a dominant frequency of 15 Hz is placed at the position $(x, z) = (2.5, 0.0)$ km. The grid increments are redefined to be $h = 5$ m in both $x$ and $z$ directions. The time sampling increment $\Delta t = 0.5$ ms; 1001 receivers with spacing 5 m are placed at a depth $z = 0.0$ km, and the virtual receivers are placed at a depth $z = 0.02$ km with spacing 5 m. Figure 2.13a shows the recorded seismograms from forward modeling. Figure 2.13b and 2.13c contains the horizontal and vertical components of the decomposed P- and S-wave seismograms.

### 2.7 Discussion

Besides the many advantages of vector decomposition, the decomposed elastic wavefield vectors contain polarization information that can be directly related to propagation angles: P-wave particle-velocity (or displacement) is parallel to the propagation direction, and S-wave particle-velocity (or displacement) is normal to the propagation direction. This provides a possible foundation for calculating propagation angles, which facilitates the subsequent procedure of obtaining angle-domain common-image gathers (ADCIGs) during RTM.

The vector decomposition methods provide complete and accurate information in the decomposed wavefields, but it comes with a price; both the attenuation and decoupled vector decomposition methods are more expensive than calculating the divergence and curl operators. The decomposed wavefields and seismograms both preserve all their original information in vector form, so they cannot be accurately processed with acoustic (one-component) processing software. A caution for using selective attenuation and decoupled propagation is
Figure 2.13. (a) is the recorded seismogram generated from the Marmousi2 model with the direct waves removed; (b) is the decomposed P-wave seismogram; (c) is the decomposed S-wave seismogram. In each pair, the left panel is the horizontal component and the right panel is the vertical component.
that both require the extrapolation model to be smoothed enough to avoid any secondary reflections or mode conversions during the extrapolation performed for decomposition, but this is already commonly used in RTM (Chattopadhyay and McMechan, 2008).

Both selective attenuation and decoupled propagation methods can be extended to viscoelastic media and 3D. However, a limitation is that both are built on the assumption of decoupled elastic wavefields, which means they are only valid for isotropic elastic media. Vector decomposition for VTI media is also possible by solving the Christoffel equation (Zhang and McMechan, 2010) at significantly greater expense. Cheaper and accurate vector decomposition methods for anisotropic media need to be explored.

A fundamental reason for doing the vector wavefield-decomposition is the potential for application in elastic reverse-time migration. In this paper, we have implemented and illustrated methods to decompose the elastic x-z wavefields during extrapolation, and to decompose x-t seismograms into P and S components. This will lead to two different strategies of implementing elastic RTMs; we can either use the original elastic seismograms as input, and decompose the wavefields during extrapolation, or use the decomposed P- and S-seismograms directly as input for elastic RTMs. Selective attenuation can potentially be used with viscoelastic migration, in which one wave mode is removed by attenuation and the other is compensated for viscoelastic losses in the same extrapolation. These, and the corresponding PP and PS imaging conditions, will be presented elsewhere.

2.8 Conclusions

Vector decomposition of P- and S-waves is superior to the traditional divergence and curl separation, as the latter damage the amplitude and phase of the input elastic wavefields. Because wavefield vector decomposition preserves all the elastic vector information, both amplitude and phase remain accurate after decomposition. Two affordable methods for isotropic elastic decomposition in the vector-domain (selective attenuation and decoupled
propagation) are implemented, illustrated and compared. Both are performed during extrapolation. Synthetic tests show that both methods give accurate decomposition results for homogeneous and smoothly varying isotropic models. The decoupled propagation is better in terms of accuracy, speed, memory requirement and numerical stability. Decomposition of the observed $x$-$t$ data into P and S vector seismograms can be achieved by downward, followed by upward elastic wavefield extrapolations through a homogeneous model with either of the wavefield-decomposition algorithms, and reconstruction of the decomposed seismograms.

2.9 Acknowledgments

The research leading to this paper is supported by the Sponsors of the UT-Dallas Geophysical Consortium. A portion of the computations were done at the Texas Advanced Computing Center. This paper is Contribution No. 1270 from the Department of Geosciences at the University of Texas at Dallas. A special thank for Dr. Jim Brown, Andrey Shabelansky and two anonymous reviewers for the useful comments and encouragements.

2.10 Appendix: Proof of equivalence of two decoupled elastic propagation algorithms

Zhang et al. (2007) proposed an algorithm for decomposition of an elastic wavefield, which is a decoupled version of Virieux (1986) staggered-grid stress-velocity formulation. In that algorithm, the P and S particle velocities are solved simultaneously. Xiao and Leaney (2010) use an additional variable ($\sigma_P$) to construct the decoupled elastic equation, which is more efficient and intuitive. The two algorithms can be proved to be mathematically equivalent as follows.

We start with a 2D version of the Xiao and Leaney (2010) decomposition algorithm, in which Virieux (1986) stress-particle velocity equations are completely preserved. Refer to equations 2.14 through 2.23.
If we add the equations 2.14 and 2.15, we obtain
\[
\frac{\partial \sigma_{xx}}{\partial t} + \frac{\partial \sigma_{zz}}{\partial t} = (2\lambda + 2\mu)(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}).
\] (2.24)

Replacing \(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\) with \(\frac{1}{\lambda + 2\mu} \frac{\partial \sigma_P}{\partial t}\), from equation 2.19, into equation 5.24, we get
\[
\frac{\partial \sigma_{xx}}{\partial t} + \frac{\partial \sigma_{zz}}{\partial t} = \frac{2\lambda + 2\mu}{\lambda + 2\mu} \frac{\partial \sigma_P}{\partial t}.
\] (2.25)

Substituting for \(\frac{\partial \sigma_P}{\partial t}\) from equation 5.25 into equations 2.20 and 2.21, the time derivatives of the P-wave \(x\) and \(z\) particle-velocity components become
\[
\frac{\partial v_{xP}}{\partial t} = \frac{1}{\rho} \frac{\lambda + 2\mu}{2\lambda + 2\mu} (\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}),
\] (2.26)

and
\[
\frac{\partial v_{zP}}{\partial t} = \frac{1}{\rho} \frac{\lambda + 2\mu}{2\lambda + 2\mu} (\frac{\partial \sigma_{xx}}{\partial z} + \frac{\partial \sigma_{zz}}{\partial x}).
\] (2.27)

With the expressions \(v_i\) (where \(i\) is \(x\) or \(z\)) and \(v_{iP}\), the subtraction form of \(v_{iS}\) in equations 2.22 and 2.23 can be rewritten
\[
\frac{\partial v_{xS}}{\partial t} = \frac{\partial v_x}{\partial t} - \frac{\partial v_{xP}}{\partial t} = \frac{1}{\rho} \frac{\lambda}{2\lambda + 2\mu} \frac{\partial \sigma_{xx}}{\partial x} + \frac{1}{\rho} \frac{\lambda}{2\lambda + 2\mu} \frac{\partial \sigma_{zz}}{\partial z} - \frac{1}{\rho} \frac{\lambda + 2\mu}{2\lambda + 2\mu} \frac{\partial \sigma_{xx}}{\partial x},
\] (2.28)

and
\[
\frac{\partial v_{zS}}{\partial t} = \frac{\partial v_z}{\partial t} - \frac{\partial v_{zP}}{\partial t} = \frac{1}{\rho} \frac{\lambda}{2\lambda + 2\mu} \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{\rho} \frac{\lambda}{2\lambda + 2\mu} \frac{\partial \sigma_{xx}}{\partial x} - \frac{1}{\rho} \frac{\lambda + 2\mu}{2\lambda + 2\mu} \frac{\partial \sigma_{zz}}{\partial z}.
\] (2.29)

The sum of \(v_{iS}\) and \(v_{iP}\) forms the complete particle-velocity component description of the S and P parts of the elastic wavefield. Thus,
\[
v_x = v_{xS} + v_{xP},
\] (2.30)

and
\[
v_z = v_{zS} + v_{zP},
\] (2.31)

which are equations 2.22 and 2.23. Equations 2.26 to 2.31 form the algorithm of Zhang et al. (2007). QED.
CHAPTER 3
VECTOR DOMAIN P AND S DECOMPOSITION IN VISCOELASTIC MEDIA∗

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3.1 Abstract

P and S decomposition is an essential step in isotropic viscoelastic reverse time migration (VRTM). Separation using divergence and curl operators doesn’t preserve the phase or amplitude of the input viscoelastic wavefield, so an alternative decomposition that preserves the vector components of the P- and S-waves in the input wavefield is desired. With a decoupled assumption, we rearrange the memory variables in the elastodynamic equations based on the standard linear solid. The vector components of the propagating P- and S-wave can be decomposed in isotropic viscoelastic media without losing accuracy. Synthetic tests show that the P- and S-waves can be decomposed with no distortion of the amplitude or phase, and have the same vector components of particle velocity that exist in the viscoelastic wavefield before decomposition.

3.2 Introduction

Anelastic effects have been widely observed in wave propagation in the Earth (Carcione et al., 1988b). To simulate viscoelastic wave propagation, the theory of linear viscoelasticity based on Boltzmann’s superposition principle has been proposed and shown to be effective (Liu et al., 1976). Carcione et al. (1988a,b); Carcione (1993) replace the time convolution in the viscoelastic constitutive equation by introducing memory variables, which allow the simulation of wavefields for models with arbitrary quality factor (Q) distributions, thus making viscoelastic computations practical and affordable in the time domain. This approach was extended to the stress-particle velocity formulation by Robertsson et al. (1994), that is used in this paper. Earth materials have been shown to have a nearly constant Q over the exploration seismic frequency range (McDonal et al., 1958; Bourbie et al., 1987). To achieve realistic simulation, Blanch et al. (1995) proposed a quick procedure for modeling constant Q behavior; Hestholm et al. (2006) combined the method of Blanch et al. (1995) and the
Nelder-Mead algorithm (Powell, 1973) and improved Q estimation. Xu and McMechan (1995)) improved the efficiency of the viscoelastic stress-particle velocity formulation by using composite memory variables, where the shear and compressional memory variables are combined by vector components to reduce RAM storage. With the linear viscoelastic formulation, attenuation loss can be compensated during both forward (source) and backward (receiver) wavefield extrapolations, which is used in true-amplitude migrations (Deng and McMechan, 2008).

Because of the superposition of P- and S-waves in all particle velocity components, the P-S separation remains a major problem in viscoelastic RTM, as well as in elastic RTM. Recently, vector domain P-S separation (vector decomposition) has shown potential value in elastic reverse time migration to get PP and PS images and obtaining ADCIGs. The decomposed P- and S-wave particle velocity vectors have better accuracy than the results from divergence and curl operators, as both phase and amplitude are preserved during the separation. The decoupled propagation (Ma and Zhu, 2003; Zhang et al., 2007; Xiao and Leaney, 2010), is now the most affordable, accurate and efficient algorithm to realize the P and S vector decomposition for an elastic isotropic wavefield. In this paper, we extend decoupled propagation for P and S decomposition to data from isotropic viscoelastic media.

3.3 Methodology

There are various methods for parameterization of viscoelastic moduli. In viscoelastic wavefield simulation algorithms with memory variables, both relaxed and unrelaxed moduli are used to characterize the model. Carcione et al. (1988a,b); Carcione (1993) use relaxed (and unrelaxed) bulk and shear moduli; Robertsson et al. (1994) use P-wave and S-wave moduli. Because of the difference in parameterizations, the attenuation effects and calculation for memory variables are also different. However, for the purpose of P and S decomposition in the vector domain, we choose to parameterize the model with P and S-wave moduli, in
which case, the memory variables can be divided into P- and S-parts. We define $M_{1r}$ and $M_{2r}$ to be the relaxed P-wave and S-wave moduli. The relations between relaxed and unrelaxed moduli, and elastic (relaxed) Lamé parameters are (Robertsson et al., 1994)

$$M_{1r} = \lambda + 2\mu, \quad (3.1a)$$

$$M_{2r} = \mu, \quad (3.1b)$$

$$M_{1u} = (\lambda + 2\mu)[1 - \sum_{l=1}^{L_p} (1 - \frac{\tau_{pl}}{\tau_{pl}})], \quad (3.1c)$$

and

$$M_{2u} = \mu[1 - \sum_{l=1}^{L_s} (1 - \frac{\tau_{sl}}{\tau_{sl}})], \quad (3.1d)$$

where $\tau_{pl}^i$ and $\tau_{sl}^i$ are stress and strain relaxation times for P-wave ($i = p$) and S-wave ($i = s$) modes for the $l$th relaxation mechanism, which are estimated through a constant Q parameterization (Hestholm et al., 2006); $L_p$ and $L_s$ are the total numbers of relaxation mechanisms for P- and S-wave moduli.

The stress-particle velocity formulation, which was initially proposed by Madariaga (1976), Virieux (1984) and Virieux (1986) to extrapolate elastic wavefields, can also be used for solving wave propagation in viscoelastic media if memory variables are added (Robertsson et al., 1994). With the parameterization of P- and S-wave moduli, Hooke’s law in a 2-D isotropic linear viscoelastic medium are (Robertsson et al., 1994)

$$\frac{\partial \sigma_{xx}}{\partial t} = M_{1u}(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}) - 2M_{2u} \frac{\partial v_z}{\partial z} + \sum_{l=1}^{L_p} e_{1l} - \sum_{l=1}^{L_s} e_{11l}, \quad (3.2a)$$

$$\frac{\partial \sigma_{zz}}{\partial t} = M_{1u}(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}) - 2M_{2u} \frac{\partial v_x}{\partial x} + \sum_{l=1}^{L_p} e_{1l} - \sum_{l=1}^{L_s} e_{22l}, \quad (3.2b)$$

and

$$\frac{\partial \sigma_{xz}}{\partial t} = M_{2u}(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}) + \sum_{l=1}^{L_s} e_{12l}, \quad (3.2c)$$
where \( v_x \) and \( v_z \) are vertical and horizontal particle velocities; \( \sigma_{xx}, \sigma_{zz} \) and \( \sigma_{xz} \) are stress components; \( e_{1l}, e_{11l}, e_{22l} \) and \( e_{12l} \) are the memory variables, which are obtained through finite-differencing

\[
\frac{\partial e_{1l}}{\partial t} = \frac{M_{1r}}{\tau_{pl}^p} \left(1 - \frac{\tau_{pl}^p}{\tau_{pl}^s}\right) \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) - \frac{e_{1l}}{\tau_{pl}^p},
\]

(3.3a)

\[
\frac{\partial e_{11l}}{\partial t} = 2 \frac{M_{2r}}{\tau_{pl}^s} \left(1 - \frac{\tau_{pl}^s}{\tau_{pl}^s}\right) \frac{\partial v_z}{\partial z} - \frac{e_{11l}}{\tau_{pl}^s},
\]

(3.3b)

\[
\frac{\partial e_{22l}}{\partial t} = 2 \frac{M_{2r}}{\tau_{pl}^s} \left(1 - \frac{\tau_{pl}^s}{\tau_{pl}^s}\right) \frac{\partial v_x}{\partial x} - \frac{e_{22l}}{\tau_{pl}^s},
\]

(3.3c)

and

\[
\frac{\partial e_{12l}}{\partial t} = \frac{M_{2r}}{\tau_{pl}^s} \left(1 - \frac{\tau_{pl}^s}{\tau_{pl}^s}\right) \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) - \frac{e_{12l}}{\tau_{pl}^s}.
\]

(3.3d)

The equations of motion are

\[
\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}\right),
\]

(3.4a)

and

\[
\frac{\partial v_z}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x}\right).
\]

(3.4b)

Equations 3.2 to 3.4 describe the viscoelastic wave propagation with P- and S-waves combined. However, in equation 3.3, the relaxed P-wave modulus \( M_{1r} \) only exists in the calculation for \( \frac{\partial e_{1l}}{\partial t} \), so \( e_{1l} \) can be regarded as the P-wave memory variable, and attenuates only the P-waves; the other memory variables \( e_{11l}, e_{22l} \) and \( e_{12l} \) are the S-wave memory variables. Following Xiao and Leaney (2010), we construct the new stress term \( \sigma_p \) which controls the propagation and attenuation only of the P-waves, with the P-wave memory variable \( e_{1l} \):

\[
\frac{\partial \sigma_p}{\partial t} = M_{1u} \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) + \sum_{l=1}^{L_p} e_{1l}.
\]

(3.5)
We define the decoupled P-wave particle velocity components to be $v_{xp}$ and $v_{zp}$; they are calculated from $\sigma_p$ by finite differencing

$$\frac{\partial v_{px}}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma_p}{\partial x},$$

(3.6a)

and

$$\frac{\partial v_{pz}}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma_p}{\partial z}.$$  

(3.6b)

This gives a decomposed viscoelastic P-wavefield (in terms of the particle velocity components); the S-wavefield can be obtained by subtracting the P-wavefield from the complete viscoelastic wavefield component-by-component, so the decoupled S-wave particle velocity components are

$$v_{sx} = v_x - v_{px},$$

(3.7a)

and

$$v_{sz} = v_z - v_{pz},$$

(3.7b)

where $v_{xs}$ and $v_{zs}$ are the horizontal and vertical particle velocity components of the S-wave.

Equations 3.2 to 3.7 provide a decoupled solution for viscoelastic wave propagations; the results are decomposed P- and S-wave particle velocity components. This approach works with any number of relaxation mechanisms, and has the same numerical stability condition as the original viscoelastic propagation equations.

### 3.4 Test on Synthetic Data

In this section, the proposed viscoelastic wavefield decomposition method is tested with data from two different viscoelastic models, together with a test on data from a homogeneous model using divergence and curl operators to demonstrate the advantages of vector
decomposition. A 2D staggered-grid finite-difference solution (Virieux, 1986) is used to solve the viscoelastic stress-particle velocity formulation of the elastodynamic equations. We use eighth-order in space and second-order in time finite-difference equations to calculate spatial derivatives at each grid point. Convolutional perfectly matched layer (CPML) absorbing boundary conditions (Komatitsch and Martin, 2007) with a width of 20 grid points inside the model are used on all four grid edges to reduce grid-edge reflections, and absorbs most of the wave energy as it approaches the model edges. Any energy which reaches the grid edge is reflected, and further attenuated as it passes again through the absorbing zone. To have sufficient grid points for eighth-order finite-differences of the derivatives in elastodynamic equations, the virtual top of the model is defined at the 5th grid point in \( z \) below the inner bound of the top absorbing layer.

The first test is performed on a homogeneous isotropic viscoelastic model with grid dimensions \( 256 \times 256 \) and grid increments \( h = 5 \) m and time increments \( dt = 0.5 \) ms. The viscoelastic model has attenuation of both P- and S-waves \( (Q_p = 30, Q_s = 40) \); Figure 3.1 shows \( 1/Q_p \) and \( 1/Q_s \) as functions of frequency. The algorithm for modeling of a constant Q can be found in Blanch et al. (1995) and Hestholm et al. (2006). A composite source, which generates both P- and S-waves simultaneously, is placed at the center of the grid. Figure 3.2a shows snapshots of the resulting viscoelastic particle velocity snapshots at five times; both P and S-waves are generated, and propagate at different velocities with different polarizations in the horizontal and vertical components.

We first separate the 2D, 2-component viscoelastic wavefield (Figure 3.2a) into divergence and curl (Figure 3.2b); the separation is good, but the original 2-component (\( x \) and \( z \)) elastic wavefield becomes one component for P and one for S. The P-wave becomes a scalar wavefield, and the S-wave become a vector wavefield with only one non-zero component; the vector component information in the input wavefield is lost.
Figure 3.1. $1/Q_p$ and $1/Q_s$ as functions of frequency. Note that both $1/Q_p$ and $1/Q_s$ are fairly constant within the seismic frequency bandwidth.

Figure 3.3a and b shows the decomposed P- and S-waves using viscoelastic decoupled propagation. Compare the snapshots with those from the undecomposed wavefield (Figure 3.2a); both vertical and horizontal components of the viscoelastic wavefields are preserved in the separations, and there are no phase or amplitude changes.

To demonstrate that the vector decomposition algorithm works in inhomogeneous as well as homogeneous viscoelastic media, a more salient test is performed on data from a part of the 2D elastic Marmousi2 model. For this test, approximate $Q_p$ and $Q_s$ values are constructed from the P- and S-wave velocity models by multiplying the P- and S-wave velocities by 30 respectively (the velocity models have the units of km/s). The original model is redefined with grid increments $h = 5$ m. The composite (P and S) source is placed at $z = 0.0$ km and $x = 2.5$ km. The time increment in the extrapolation is $dt = 0.5$ ms. Figure 3.4a shows a snapshot of the elastic particle velocity wavefield components before decomposition.
Figure 3.2. a) Snapshots of 2D wave propagation in a viscoelastic homogeneous medium with a composite P-S source at the center. Horizontal and vertical components of the wavefields are recorded. b) Separation results with the divergence (upper panel) and the curl (lower panel) operators. Note the large differences in amplitudes between (a) and (b).
Figure 3.3. Separated snapshots of wave propagation using viscoelastic decoupled propagation. (a) decomposed vector P-wavefield; (b) decomposed vector S-wavefield.

Figure 3.4b and c are the decomposed P- and S-wavefields respectively using the viscoelastic decoupled algorithm. The results shows acceptable decomposed P- and S-waves in both vertical and horizontal components.

3.5 Discussion

The viscoelastic vector decomposition method provides complete and accurate information in the decomposed viscoelastic wavefields, and thus can potentially be used in true-amplitude viscoelastic RTM, and in both PP and PS ADCIGs with a vector-based image
Figure 3.4. Snapshots of the wave propagation in a part of the elastic Marmousi2 model, using the smoothed P-wave velocity model as the background with wavefield snapshots overlapped with 50% transparency. The $Q_p$ and $Q_s$ values are the $V_p$ and $V_s$ values multiplied by 30 respectively. All snapshots are particle velocity components; the left column is $v_x$ and the right column is $v_z$. (a) is the complete elastic particle velocity wavefield without P and S decomposition; (b) is the decomposed P-wavefield components; (c) is the decomposed S-wavefield components.
condition. A prerequisite is that all the parameter models (Vp, Vs, Qp, Qs and density) need to be smoothed to do the decomposition so there are no extra reflected or converted waves generated during the extrapolations (which is also a prerequisite in RTM to reduce migration artifacts). Viscoelastic RTM also requires compensation of attenuation of Q during the back propagation of the receiver wavefield (e.g., Deng and McMechan, 2008), which is beyond the scope of the present study.

The decomposition algorithm is easily extended to 3D, in which the P-wave stress $\sigma_p$ (equation 3.5) remains a scalar, but is calculated using three particle velocity components. P and S vector decomposition in anisotropic media is possible, but expensive to calculate (Zhang and McMechan, 2010; Cheng and Fomel, 2014).

3.6 Conclusions

The isotropic vector-domain P- and S-wave decoupled vector decomposition algorithm is extended to viscoelastic wavefields, and is superior to divergence and curl operators as it preserves both the elastic amplitude and phase information. Synthetic tests show that the proposed decoupled viscoelastic vector decomposition method gives accurate decomposition results for data from inhomogeneous, isotropic, viscoelastic models.

3.7 Acknowledgements

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CHAPTER 4
VECTOR-BASED ELASTIC REVERSE TIME MIGRATION*

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4.1 Abstract

Prestack elastic reverse-time migration (RTM) of multicomponent seismic data requires separating PP and PS reflections before, or as part of, applying the image condition, and using image conditions that preserve both angle and amplitude information. Both of these requirements are best achieved when all operations are on vectors. We create a new 2D migration context for isotropic, elastic RTM which includes decomposition of the elastic source and receiver wavefields into P- and S-wave vectors by decoupled elastodynamic extrapolation which retains the same stress and particle velocity components as the input data. Then, propagation directions of both incident and reflected P- and S-waves are calculated directly from the stress and particle velocity definitions of the P- and S-wave Poynting vectors. An excitation-amplitude image condition that scales the receiver wavelet by the source vector magnitude produces angle-dependent images of PP and PS reflection coefficients with the correct polarities, polarization, and amplitudes. It thus simplifies the process of obtaining PP and PS ADCIGs; it is less effort to generate ADCIGs from vector data than from scalar data. Examples show that the resulting prestack elastic images are nearly identical to the corresponding source-normalized crosscorrelation images, and have improved resolution, because the wavelet broadening that results from the crosscorrelation is not present.

4.2 Introduction

Migration of multicomponent data has previously been attempted using both ray-based and wave-based solutions. Examples of ray-based solutions include the multicomponent Kirchhoff migrations of Kuo and Dai (1984), Dai and Kuo (1986), and Hokstad (2000), who calculate PP and PS traveltimes and sum data along their corresponding traveltime trajectories; the P and S separation is done implicitly by using P- and S-velocity models to compute the traveltimes. Multicomponent elastic Kirchhoff migration has the same limitations as acoustic Kirchhoff migrations; ray theory no longer applies if the geology becomes
complicated (Gray et al., 2001). Wave-based solutions (Chang and McMechan, 1986, 1994; Whitmore, 1995) backpropagate the elastic data wavefields with the elastic wave equation (Wapenaar and Haimé, 1990) and have fewer limitations than ray-based migrations; wave, rather than ray, extrapolations are needed to get reliable image amplitudes.

 Prestack elastic RTM, as a wave-based solution for imaging common-source data, constructs the source elastic wavefields forward in time, and reconstructs the receiver elastic wavefields propagating backward in time by using multicomponent seismic data as boundary conditions (Chang and McMechan, 1986, 1994; Nguyen and McMechan, 2015). The two main difficulties in imaging with elastic wave or elastodynamic equations are P and S wavefield separation and deriving physically correct image conditions. For P and S separation, the polarization of surface data alone is not sufficient to separate the wavefield accurately (Pestana et al., 1989); Sun (1999) downward extrapolates the elastic wavefield from the acquisition surface using the elastic wave equation, and then separates the wavefields into P- and S-waves using divergence and curl operators (Aki and Richards, 1980), followed by independent upward extrapolation of the separated P-and S-wave modes back to the acquisition surface using acoustic wave equations. However, the spatial derivatives in the divergence and curl operators introduce a 90° phase shift and also changes the amplitude in the original elastic wavefield (Sun et al., 2001, 2004, 2011). Thus velocity-dependent corrections are required to obtain accurate angle-dependent image amplitudes (Sun et al., 2011; Nguyen and McMechan, 2015).

 The image condition strongly influences the quality of the migrated images. Yan and Sava (2008a) review various elastic image conditions, including imaging with vector displacement components and imaging with scalar and vector potentials from Helmholtz decomposition, and conclude that the potential-based image condition gives clearer PP and PS images than directly crosscorrelating vector displacements. However, crosscorrelation of vector components does not preserve the wavefield amplitudes in a form that gives migrated images that
can be interpreted as reflection coefficients. To overcome this limitation, we consider a de-convolution type of image condition that is based on an image time defined as the time at which the source amplitude maximum occurs at each grid position (Botelho and Stoffa, 1988; Loewenthal and Hu, 1991). This approach later evolved into the acoustic and elastic excitation-amplitude image conditions (Nguyen and McMechan, 2013, 2015), which we use below.

We create a new workflow for prestack elastic RTM with P- and S-wave vector decomposition. Different from ‘wavefield separation’ by divergence and curl operators, P- and S-wave decomposition preserves the vector components in the decomposed P- and S-waves (Zhang and McMechan, 2010). Elegant algorithms are available to achieve this P and S-wave decomposition in isotropic elastic media (Ma and Zhu, 2003; Zhang et al., 2007; Xiao and Leaney, 2010). The decomposed P- and S-waves have the same components as the input elastic data; no phase or amplitude changes occur during the vector decomposition.

For migration, a corresponding image condition also needs to be developed to use vector data as input to produce separate PP, PS, SS and SP images with physically correct amplitudes; below we consider only PP and PS images. Nguyen and McMechan (2013) compare the crosscorrelation image condition with the excitation-amplitude image condition, and show that the latter produces higher resolution images because only the peak values in the source wavefield are used, and it also doesn’t have the large storage requirement and high I/O burden of the crosscorrelation image condition. Non-source-normalized crosscorrelation also has the drawback of generating images whose amplitudes have no physical meaning (Chattopadhyay and McMechan, 2008). Based on the decomposed P- and S-wave vector information and extending the excitation-amplitude image condition from acoustic to elastic, we formulate a new vector-based prestack image condition which gives both accurate subsurface structural geometry information and accurate PP and PS reflectivities, without any amplitude or phase corrections being needed, provided that transmission and attenuation losses during extrapolation are correctly compensated.
Angle-domain common-image gathers (ADCIGs) (de Bruin et al., 1990) can also be efficiently obtained using the vector-based prestack image condition, which we also implement and illustrate below. Poynting vectors (Červený, 2001; Dickens and Winbow, 2011; Yoon et al., 2011; Zhao et al., 2012; Vyas et al., 2011) are inherently ideal for obtaining ADCIGs during elastic RTM, except where waves of the same type (P or S) overlap in slowness and location. Dickens and Winbow (2011) use particle-velocity and stress components to calculate Poynting vectors. Below, we employ the novel approach of directly using the decomposed P and S stress and particle-velocity vector components to calculate Poynting vectors to get the propagation directions of P- and S-waves separately; then the image values are sorted by incident angle to generate PP and PS ADCIGs.

We begin by showing decoupled elastic wavefield extrapolation with vector PP and PS wavefield decomposition. Then we illustrate the procedure to obtain propagation directions from decomposed PP and PS vector wavefields, followed by implementation of the vector-based prestack image condition, and the methodology to generate PP and PS ADCIGs. To limit the scope, we omit anisotropy and viscosity. Finally, we illustrate this elastic wavefield imaging procedure with multicomponent data for a flat layered model, and for a portion of the elastic Marmousi2 model. The equations and examples are all in 2D, but all the concepts can be extended to 3D.

4.3 Methodology

Figure 4.1 contains the flowchart of elastic RTM with the vector-based prestack image condition. The source wavefield extrapolation is done before the receiver wavefield extrapolation, and both extrapolations involve P and S wavefield decomposition in the vector domain (Wang et al., 2015). The decomposed P- and S-wave particle-velocity components and stresses are used to obtain their propagation directions (Dickens and Winbow, 2011).
Figure 4.1. Flowchart of elastic RTM with the vector-based prestack image condition. The source wavefield extrapolation is done before the receiver wavefield extrapolation, and both extrapolations include P and S wavefield decompositions in the vector domain. The decomposed P- and S-wave particle-velocity and stress vectors are used to obtain their propagation directions via Poynting vectors. Because of the different polarizations of PP and PS reflections, the procedures for determining reflection signs in the image condition are also different.

Because of the different polarizations of PP and PS reflections, the procedures to determine the reflection’s signs are also different. In the following subsections, the procedures are explained in detail for each.
4.3.1 Elastodynamic extrapolation

Madariaga (1976) and Virieux (1984, 1986), define a 2D stress-particle-velocity formulation for 2D elastic isotropic media:

\[
\frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z},
\]

(4.1)

\[
\frac{\partial \tau_{zz}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x},
\]

(4.2)

\[
\frac{\partial \tau_{xz}}{\partial t} = \mu \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right),
\]

(4.3)

\[
\rho \frac{\partial v_x}{\partial t} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},
\]

(4.4)

and

\[
\rho \frac{\partial v_z}{\partial t} = \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x},
\]

(4.5)

to solve the elastodynamic equations with second-order in space, and second-order in time, staggered-grid finite-differencing. In equations 5.24-5.23, \(\lambda\) and \(\mu\) are Lamé parameters, \(\tau_{xx}\) and \(\tau_{zz}\) are horizontal and vertical normal stresses, \(\tau_{xz}\) is shear stress, and \(v_x\) and \(v_z\) are horizontal and vertical particle-velocity components. Unlike the displacement finite-difference wave equation formulation (e.g., Kelly et al., 1976), the stress-particle-velocity elastodynamic formulation on a staggered grid is stable for all values of Poisson’s ratio (Levander, 1988), and is accurate for modeling irregular fluid-solid interfaces. \(\lambda\), \(\mu\) and \(\rho\) are not uniquely defined by the P- and S-propagation velocities; at least one more piece of independent information is needed, which could be from locally-determined, empirical velocity-density relations, or well logs, or petrophysical measurements on cores, or elastic full wavefield inversion.

The stress-particle-velocity formulation is used to extrapolate both source and receiver wavefields in the elastic RTM; provided that the propagation times are sufficiently large that all the wavefield values at the largest times are very small, we need to record only the particle
velocities as seismograms, because the stress components are retrieved iteratively from the particle velocities during extrapolation (Nguyen and McMechan, 2015). If the magnitudes of the stresses and particle velocities are too large at the grid boundaries, then saving and inserting only particle velocities will generate large grid-edge artifacts; this can be overcome with added computational expense by saving both stresses and particle velocities (Nguyen and McMechan, 2015).

4.3.2 P and S decomposition

P and S decomposition is necessary to get PP and PS images that are free from cross-talk artifacts. The stress-particle-velocity formulation extrapolates elastic wavefields with the P- and S-waves superimposed and coupled across the wavefield components, so if they are not separated, either before extrapolation, or as part of the image condition, they will be superimposed in the migrated images. Instead of using curl and divergence, we decompose the wavefield in the vector domain, so the vector information in the input elastic wavefield is preserved in the output. This can be achieved by calculating an additional P-wave stress \( \tau^p \), which is a scalar wavefield similar to the pressure in the acoustic wave equation, while solving the stress-particle-velocity formulation in equations 5.24-5.23. The calculation for the P-wave stress has a scalar form (Xiao and Leaney, 2010)

\[
\frac{\partial \tau^p}{\partial t} = (\lambda + 2\mu)(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}).
\]  

(4.6)

Then, the vertical and horizontal particle-velocity components of the P-waves are calculated from \( \tau^p \) by finite differencing

\[
\frac{\partial v^p_z}{\partial t} = \frac{1}{\rho} \frac{\partial \tau^p}{\partial z},
\]  

(4.7)

for the vertical component \( v^p_z \) of the P-wave particle-velocity and

\[
\frac{\partial v^p_x}{\partial t} = \frac{1}{\rho} \frac{\partial \tau^p}{\partial x},
\]  

(4.8)
for the horizontal component $v_x^p$ of the P-wave particle-velocity. This gives a complete description the particle-velocity components of the P-wavefield; the S-wavefield can then be obtained by a simple direct subtraction of the P-wavefield from the complete wavefield, component-by-component;

$$v_z^s = v_z - v_z^p,$$  \hspace{1cm} (4.9)

and

$$v_x^s = v_x - v_x^p,$$  \hspace{1cm} (4.10)

where $v_z^s$ and $v_x^s$ are vertical and horizontal particle-velocity components of the S-waves.

The process of decomposition is embedded in the wavefield extrapolation, which is a part of the prestack elastic migration. The decomposed results ($v^p$ and $v^s$) are P-wave and S-wave particle-velocity vectors. Zhang et al. (2007) use a different formulation, but they can be proven to be mathematically equivalent (See Appendix A of Wang et al., Wang et al. (2015).

The vector decomposition algorithm (equations 4.6-4.10) assumes decoupling of P- and S-waves, so to implement this in the elastic RTM, the parameter model needs to be smoothed to avoid artifacts caused by secondary reflections and mode conversions produced by coupling at reflectors. The amount of smoothing required is model dependent. Too much smoothing will, to some extent, affect the image amplitudes and angles and may turn wide-angle reflections into turned waves (Nguyen and McMechan, 2015); too little smoothing will retain artifacts from secondary reflections. The length of the optimal smoothing operator is the order of the dominant wavelength; discussions and examples are given by Chattopadhyay and McMechan (2008) and Wang et al. (2015). In the present context, the source wavefield ideally contains only P-waves, and the reconstructed receiver wavefield contains both PP and PS converted waves, so P and S decomposition via equations 4.6-4.10 is required for the receiver wavefield extrapolation, and is also recommended for the source wavefield extrapolation to reduce any
new artifacts produced during the extrapolation when calculating propagation directions. It is not sufficient to do acoustic (P-wave) extrapolation only, rather than elastic, for the source wavefield, because acoustic and elastic amplitudes are not the same (e.g., Deng and McMechan, 2008) and scalar acoustic waves also do not provide the vector components needed for the angle calculations.

4.3.3 Elastic vectors and propagation directions

Most existing methods for calculating propagation directions for obtaining ADCIGs from scalar data involve complicated wavefield propagation angle decompositions (Yan and Xie, 2011; Jin et al., 2014), or require that the migrated PP images provide reflector normal directions (Zhang and McMechan, 2011) as part of ADCIG extraction. The power of the readily-available polarization and elastic vector component information is previously underestimated. In 2D elastic isotropic media, the P-wave particle-velocity vectors are polarized parallel to the propagation direction, and the S-wave particle-velocity vectors are polarized perpendicular to the propagation direction in the 2D plane. This relation is useful only if the elastic wavefield is completely decomposed into P- and S-wave vectors, so the propagation directions can be obtained from the P and S polarizations. However, the polarization gives two possible propagation directions, whereas the Poynting vectors, which involve both particle-velocity and stress, give only one direction.

By definition, the Poynting vector \( \mathbf{s} \) is the energy flux density vector (Červený, 2001; Yoon and Marfurt, 2006) and defines the energy flow direction; its magnitude is the amount of energy transmitted through a unit area per unit time, and the angles of the Poynting vectors can be used for angle gather calculation (Dickens and Winbow, 2011; Yoon et al., 2011; Zhao et al., 2012; Vyas et al., 2011).

\[
\mathbf{s}_j = -\tau_{jk}v_k, \quad (4.11)
\]
where \( j \) and \( k \) indicate the \( x \) or \( z \) component (in 2D) of the Poynting vectors \( s \), which is consistent with the notation of Dickens and Winbow (2011); \( \tau_{jk} \) is the stress tensor, and \( v_k \) is the \( k \)th component of the particle-velocity. Since we use the stress-particle-velocity formulation for extrapolation, vector components of both stress and particle-velocity are available for calculating the Poynting vectors. Equation 5.9 does not distinguish between P- and S-waves; both are implicitly included. However, the P and S vector wavefield decomposition makes separate calculations of P and S Poynting vectors possible. The propagation of P-waves involves only the normal stress, so we set \( j = k \) in equation 5.9, and the Poynting vectors for P-waves is

\[
    s^p_j = -\tau^p v^p_j, 
\]

where \( v^p_j \) are components of decomposed P-wave particle-velocity; \( \tau^p \) is the P-wave stress from the decomposition algorithm (equations 4.6-4.10), and \( s^p_j \) is a component of the P-wave Poynting vector with subscript \( j \) as the component index.

We subtract the scalar P-wave stress \( \tau^p \) from the normal stress components \( \tau_{kk} \) (only for \( k = x \) or \( z \)) in the complete stress tensor \( \tau_{jk} \); the remaining non-zero elements are the components of the S-wave stress tensor which can be viewed as the stress version of equations 4.9 and 4.10. From the Poynting vector formulation in equation 5.9, the components of the S-wave Poynting vector can be obtained as

\[
    s^s_j = -[\tau_{jk} - \tau^p \delta(j - k)] v^s_k, 
\]

where \( v^s_k \) is the \( k \)th component of the decomposed S-wave particle-velocity vector, and \( \delta \) is the Dirac delta function. Equation 5.10 will later be used to obtain reflector normal and incident angles during RTM, and equation 5.11 can potentially be used to get PS reflection angles.

Consider a 2D test to demonstrate why vector wavefield decomposition is essential to predict propagation directions, and to show the result of the above decomposition algorithm. In
Figure 4.2. (a) Horizontal and (b) vertical particle-velocity components of an elastic wavefield from P- and S-wave sources at different locations contain superimposed P- and S-wave fronts. In a homogeneous model, we place a P-wave source and an S-wave source at different positions, and a snapshot of particle velocities is captured at a later time (Figure 4.2). Inside the black box of Figure 4.2, the P and S particle-velocity components are superimposed, and the relations between particle motion and propagation direction break down; the corresponding Poynting vectors give only the composite (superimposed) P and S energy flow direction (Figure 4.3a and 4.3b), not the individual P- and S-wave propagation directions. After P- and S-wave vector decomposition (Figure 4.3c and 4.3e), the particle velocities contain opposite directions in alternate adjacent half cycles, and so still do not define their propagation directions. Using the decomposed P and S particle-velocity and stress components, with equations 5.10 and 5.11, produces both P- and S-wave Poynting vectors, which point in only one (propagation) direction at each grid point for both P- and S-waves (Figure 4.3d and 4.3f).
Figure 4.3. Enlarged image of the vector sum of the particle-velocity components in the boxed area in Figure 4.2a and 4.2b. The arrows indicate the particle-velocity vector directions in the superimposed P and S wavefronts in (a), and the corresponding Poynting vectors in (b), which give the energy flow directions of the superimposed P- and S-waves. After P and S vector wavefield decomposition, the particle velocity polarization directions (in c and e) along with the stress vectors produce Poynting vector directions in (d) for the decomposed P-waves and in (f) for the decomposed S-waves. The Poynting vectors are calculated by equations 5.10 and 5.11, for the P- and S-waves, respectively.
4.3.4 Vector-based elastic image condition

The P and S vector wavefield decomposition and Poynting vectors provide the basis for constructing a vector-based elastic image condition, which is an extension of the excitation-amplitude image condition (Nguyen and McMechan, 2013, 2015) and generates PP and PS images directly from the decomposed elastic wavefield vectors. The source wavefield extrapolation needs to be done prior to the reverse-time receiver wavefield extrapolation. During the source wavefield extrapolation, the image time \( T \) is determined at each grid point \((x, z)\) as the one-way time from the source to that grid point and corresponds to the maximum particle-velocity amplitude at \((x, z)\) during the source wavefield extrapolation over all times. Thus,

\[
T(x, z) = \text{argmax}\{A(x, z), t\}, \tag{4.14}
\]

where \( A \) is the amplitude of the source wavefield, and the argmax operator gives values of time \( t \) corresponding to the maximum value of \( A \) at \((x, z)\) over all times. The image time \( T(x, z) \) determines when to apply the image condition at \((x, z)\) during the reverse-time receiver wavefield extrapolation to produce images.

Several modifications need to be made to the excitation amplitude image condition to be applicable for imaging directly using vectors. First, a vector magnitude is used to represent the source amplitude

\[
A = \|v^p_{\text{src}}\|, \tag{4.15}
\]

where \( v^p_{\text{src}} \) is the source P-wavefield particle-velocity vector, and the \( \| \cdot \| \) operator calculates the magnitude of the vector argument. Second, in addition to the image time \( T \), the P-wave particle-velocity vector components of the source wavefield

\[
V^p_{\text{src}}(x, z) = v^p_{\text{src}}(x, z, T) \tag{4.16}
\]
need to be saved at the time $T$ when the maximum amplitude is reached at each $(x, z)$ location. The corresponding P-wave stress (via equation 4.6)

$$\Gamma^p(x, z) = \tau^p(x, z, T).$$

(4.17)

is also saved for calculating the propagation direction using the Poynting vector. The P-wave stress $\tau^p$ has a scalar form and so the saved stress $\Gamma^p$ also has only one value at each grid point. The reason for saving both $V_{src}^p$ and $\Gamma^p$ is that both are necessary for generating Poynting vectors using equation 5.10 (Dickens and Winbow, 2011), and $V_{src}^p$ is also needed for determining the sign of the reflection coefficient in the following procedure; saving only the maximum amplitude and propagation direction is not sufficient.

During the source wavefield extrapolation, the image time map $T(x, z)$, the P-wave particle-velocity vector $V_{src}^p$, and the P-wave stress $\Gamma^p$ (only at the maximum magnitude at each grid point), are stored in RAM. Compared to the crosscorrelation image condition, which requires full source wavefield snapshots at all time steps, the storage requirement for this vector image condition is orders of magnitude lower (Nguyen and McMechan, 2013, 2015).

The reverse-time receiver wavefield extrapolation is performed after the source wavefield extrapolation is completed. The receiver wavefield is also decomposed into P- and S-wave vectors at each time step. The image at each grid point is obtained at its image time $T(x, z)$. The incident angle in this paper, is evaluated with P-wave Poynting vectors from both source and receiver decomposed P-wave particle-velocity and stress via equation 5.10; an alterative is to use reflector normals provided from previously migrated images (Zhang and McMechan, 2011). To make the values of the images physically meaningful, we need to get the angle-dependent reflectivity information from the image condition; the most straightforward method is to use the magnitude of the reconstructed receiver particle-velocity wavefield divided by that of the source wavefield. However, magnitudes are always positive, so we also
need to determine the corresponding sign of the reflection coefficient. Thus, the application of the vector-based prestack image condition is performed in two steps as described in the following two subsections.

**Step 1. Calculating the reflector normal, incident angle and sign of PP and PS reflection coefficients at each reflection position.**

The signs of both PP and PS reflections can be determined from the principle that the incident and the reflected waves have the same polarity for a negative reflection coefficient, and opposite polarity for a positive reflection coefficient (Aki and Richards, 1980). For a PP reflection in an isotropic medium, the incident angle equals the reflection angle. At the image time \( T \) at each grid point, the reflector normal direction \( \mathbf{n} \) at this grid point (on the same side of the reflector as the incident and reflected waves) can be obtained from the PP reflection.

\[
\mathbf{n}(x, z) = \tilde{s}^p_{rec}(x, z, T) - \tilde{S}^p_{src}(x, z),
\]

(4.18)

where \( \tilde{S}^p_{src}(x, z) \) is the source P-wave Poynting vector calculated using equation 5.10 with \( V^p_{src} \) and \( \Gamma^p \) as input, and \( \tilde{s}^p_{rec}(x, z, T) \) is the receiver P-wave Poynting vector at image time \( T \). The \( \sim \) symbol above \( S \) and \( s \) indicate normalization to a unit vector. The incident angle is then calculated as

\[
\theta = \frac{1}{2} \cos^{-1}[-\tilde{S}^p_{src}(x, z) \cdot \tilde{s}^p_{rec}(x, z, T)],
\]

(4.19)

which is the same for both PP and PS reflections, and thus can be used as the reference for generating both PP and PS ADCIGs. Similarly, PP and PS reflection angles and corresponding common-image gathers can also be generated with geometrical relations between the reflection normal (equation 4.18), the P-wave Poynting vectors (equation 5.10) and the S-wave Poynting vectors (equation 5.11), but only the incident angle (\( \theta \) in equation 4.19) common-image gathers are calculated and shown in the examples below.

A basic assumption for determining the reflection signs is that the phase change due to the reflection is either 0 or \( \pm \pi \) radians. Thus, this procedure, as currently implemented, is
more reliable for precritical than postcritical reflections, as large phase changes occur in the latter.

The polarity of a PP reflection can be determined from the components of the incident and reflected P-wave particle-velocity normal to the reflector (Figure 4.4a), each determined separately in their respective propagating wavefields. At a reflection point, the projection of the source P-wave particle-velocity vector $V_{src}^p(x, z)$ to the reflector normal direction $n$, gives the projected vector $\hat{V}_{src}^p$. During the receiver wavefield extrapolation, at the image time of each grid point, the projection of the P-wave particle-velocity vector to the reflector normal direction $n$ gives $\hat{v}_{rec}^p$. The sign of the PP reflection at each $(x, z)$ grid point can be then obtained from the dot product of these two vectors.

$$\text{sgn}_{pp}(x, z, \theta) = \begin{cases} +1, & \text{if } \hat{V}_{src}^p(x, z) \cdot \hat{v}_{rec}^p(x, z, T) < 0, \\ -1, & \text{if } \hat{V}_{src}^p(x, z) \cdot \hat{v}_{rec}^p(x, z, T) > 0. \end{cases}$$ (4.20)

The calculation does not have to be very accurate as it is only a sign determination.

The polarity of a PS reflection is determined by the component of the S-wave particle motion projected parallel to the reflector (Figure 4.4b). Similar to the procedure to get the PP reflection signs, we project $V_{src}^p(x, z)$ and the receiver S-wave particle-velocity vector $v_{rec}^s(x, z, T)$ to the reflector tangent to get their projections $\hat{V}_{src}^p$ and $\hat{v}_{rec}^s$, and the sign of the PS reflection can be obtained by

$$\text{sgn}_{ps}(x, z, \theta) = \begin{cases} +1, & \text{if } \hat{V}_{src}^p(x, z) \cdot \hat{v}_{rec}^s(x, z, T) < 0, \\ -1, & \text{if } \hat{V}_{src}^p(x, z) \cdot \hat{v}_{rec}^s(x, z, T) > 0. \end{cases}$$ (4.21)

Inaccurate velocity models will lead to a spatial shift of the sign distributions as well as the corresponding magnitudes, which is also present in all other imaging conditions.

**Step 2. Calculating reflectivities of PP and PS reflections as a function of position and incident angle.**
Figure 4.4. Kinematic example of (a) PP and (b) PS particle motions and the corresponding reflection signs. \( \tilde{S}^i_{\text{src}} \) and \( \tilde{S}^i_{\text{rec}} \) are propagation directions of the vector source (src) and receiver (rec) wavefields, where \( i \) is the wave mode (P or S); \( n \) is reflector normal determined from the incident and reflected P-waves; \( V^i_{\text{src}} \) and \( v^i_{\text{rec}} \) are particle-velocity vectors; \( \hat{V}^p_{\text{src}} \) and \( \hat{v}^p_{\text{rec}} \) are incident and reflected P-wave particle-velocity vectors projected onto the reflector normal; \( \hat{V}^p_{\text{src}} \) and \( \hat{v}^s_{\text{rec}} \) are the projections, of the incident P-wave and reflected S-wave particle-velocity vectors, tangent to the reflector.
With knowledge of the signs of the PP and PS reflections, the calculation of PP and PS reflection coefficients is straightforward. At the image time of each grid point, the magnitude of the receiver (reflected) P- or S-particle-velocity divided by the maximum (peak or trough) source (incident) magnitude is the absolute value of reflectivity at angle $\theta$ (Sheriff and Geldart, 1995), and is multiplied by the sign of the PP or PS reflection to get the corresponding signed reflectivity.

$$r_{pp}(x, z, \theta) = \text{sgn}_{pp}(x, z, \theta) \frac{\|v^p_{rec}(x, z, T)\|}{\|V^p_{src}(x, z)\|},$$  

(4.22)

and

$$r_{ps}(x, z, \theta) = \text{sgn}_{ps}(x, z, \theta) \frac{\|v^s_{rec}(x, z, T)\|}{\|V^p_{src}(x, z)\|},$$  

(4.23)

where $r_{pp}$ and $r_{ps}$ are PP and PS reflection coefficients, respectively. There is no possibility of instability associated with division by a small number because $V^p_{src}(x, z)$ is the maximum P-wave source amplitude at each $(x, z)$. It is also possible to calculate multipath contributions by saving more than one image time at each grid point; see the sparse crosscorrelation implementation by Nguyen and McMechan (2015).

In step 1, we constrain the reflection sign calculation to only precritical reflections. However, the amplitudes of both source and receiver wavefields are accurate even for critical and postcritical reflections, which makes the absolute values of the $r_{pp}$ and $r_{ps}$ correct. Unwrapped phase can also be accurately extracted for any wavelet (cf. Zhu and McMechan (2011)), but propagation angles cannot be, where the phase is changing near and beyond a critical angle. The problem comes from the changing orientation of the apparent phase velocity vector when the position of a peak in the wavelet changes location progressively within the wavelet as a function of propagation distance [cf. Figure 5 of McMechan (1983b)]. The apparent phase and group velocities are not the same when the reflection coefficient goes complex.
4.3.5 Image condition summary

In summary, the vector-based prestack image condition is implemented by a forward source wavefield propagation to extract the required wavefield attributes and time information, followed by application of the image condition during a backward extrapolation of the receiver wavefield. At each grid point, during the source wavefield extrapolation, the maximum amplitude of the particle-velocity vector is detected, and the corresponding time (the image time $T$), the P-wave particle-velocity (vector components) and the P-wave stress (scalar) are saved. Then, during the reverse-time receiver wavefield propagation, the particle-velocity reflection amplitude at the image time $T$ at each grid point is divided by the previously saved maximum source particle-velocity amplitude at the same time and location, and multiplied by the reflection sign determined from source/receiver and PP/PS propagation directions and polarizations (equations 4.22 and 4.23). The division is a deconvolution image condition, and the migration of a single shot produces the angle dependent reflection coefficients; see Nguyen and McMechan (2013) for the corresponding acoustic version.

With multiple sources, the image values can be sorted to form angle-domain common-image gathers for subsequent analyses such as amplitude versus angle (AVA) analysis and migration velocity analysis (MVA). However, to obtain accurate angle dependent reflectivities, compensations of attenuation and transmission losses are also necessary during the source and receiver wavefield extrapolations of the prestack RTM (de Bruin et al., 1990; Deng and McMechan, 2007, 2008), which are difficult to implement. In this paper, we concentrate only on the image conditions.

4.4 Synthetic Tests

We test the vector-based prestack image condition with two synthetic elastic data sets; the first is for a flat layered model, and the second is for a portion of the Marmousi2
model (Martin et al., 2002). Both simulations use an eighth-order in space, second-order in
time, stress-particle-velocity, staggered-grid, finite-difference solution of the isotropic elasto-
dynamic equation (Madariaga, 1976; Virieux, 1986; Levander, 1988). Convolutional perfectly
matched layer (CPML) absorbing boundary conditions (Komatitsch and Martin, 2007) are
used on all four grid edges to reduce unwanted reflections; the CPML code is downloaded
and modified from the Komatitsch (2007) public domain website. To have sufficient grid
points for eighth-order finite-differences of the derivatives in elastic wave equations, the vir-
tual surface ($z = 0$) is defined at the 5th grid point on z-axis from the inner bound of
top absorbing layer. The sources for both tests are explosive Ricker wavelets with 15 Hz
dominant frequency.

4.4.1 Layered model test

Consider the 2D flat layered model (Figure 4.5a), which has 5 m grid spacing in both
z- and x-directions. The source is located at $(x, z) = (2.0, 0.0)$ km; 801 receivers are evenly
spaced along the surface ($z = 0.0$ km) from 0.0 km to 4.0 km, with 5 m spacing; the time
sample increment is 0.5 ms. The source is recorded by all receivers. We record the x and z
particle velocities as 2-component synthetic seismograms; Figure 4.5b and 4.5c contain the
representative 2-component common-source gather for the source at $x = 2.0$ km, with the
direct waves removed.

The parameter models ($v_p$, $v_s$ and density) need to be smoothed before migration. In
the prestack elastic RTM for each common-source gather, the source wavefield is first ex-
trapolated to record the image time map $T$ (Figure 4.6a), the P-wave stress $\Gamma_p$ (Figure 4.6b)
and the two components of particle-velocity vector $V_{p_{src}}$ (Figure 4.6c and 4.6d) at the time
of the maximum magnitude of the particle-velocity vector at each grid point. The vector-
based elastic image conditions (equations 4.22 and 4.23) are applied during the reverse-time
decoupled extrapolation of the receiver wavefield; Figure 4.7 contains the migrated PP and
Figure 4.5. (a) Three-reflector model; the red spot is the source location, and the blue squares are every 35th receiver. (b) and (c) are the horizontal and vertical particle-velocity seismograms, respectively, with the direct waves removed. Both components contain both P- and S-waves.
PS images from the elastic common-source gather in Figure 4.5b and 4.5c. Figure 4.7a and 4.7b contains the migrated PP and PS images obtained using the vector-based image condition, and for comparison, Figure 4.7c and 4.7d contain the normalized crosscorrelation images obtained using divergence and curl for P and S separation (Sun and McMechan, 2001; Nguyen and McMechan, 2015). In the context of the vector-based image condition, the sign of the reflection is determined using both polarization and propagation directions, rather than polarization alone, thus the PS image (Figure 4.7b) doesn’t have the polarity reversal problem which exists in images using the crosscorrelation image condition (Figure 4.7d); therefore, the migrated common-source PS images from the vector-based migration will stack constructively.

The peak values in the prestack migrated images from the vector-based image condition (Figure 4.7a and 4.7b) can be directly used as reflection coefficients, and this is also true for the normalized crosscorrelation (Figure 4.7c and 4.7d) when amplitude compensations are included (Nguyen and McMechan, 2015), and phase reversal of the PS image is corrected by multiplying its amplitudes by $-1$ in the region to the left of the source (Sun and McMechan, 2001); the phase changes caused by spatial derivatives in divergence and curl operators will be the same for both source and receiver wavefields and so will have no net effect during crosscorrelation or normalized crosscorrelation; thus no other phase compensation is needed in this case.

We illustrate the behavior of the prestack image conditions in the precritical region by extracting the migrated image values with maximum absolute value in three depth windows containing the migrated images of the three reflectors; the close correspondence of the black and red lines in Figure 4.8 indicate equivalent accuracy of the vector-based and normalized crosscorrelation image conditions in approximating reflection coefficients. This is based on the condition that no attenuation or transmission losses occur, and that geometrical spreading is automatically and accurately compensated during the elastodynamic extrapolations of
Figure 4.6. (a) is the image time map; (b), (c) and (d) are the P-wave stress map, and the horizontal and vertical particle-velocity component maps, respectively; all correspond to the maximum magnitude of source P-wavefield particle-velocity vector over all times at each grid point. Since the model is smoothed, the P-wave stress values are all at nearly the same phase (the peak or trough); as a result, the values of the P-wave stress have the same sign at all grid points.

the source and receiver wavefields. To satisfy these conditions, only the precritical PP and PS image data from the uppermost reflector are compared with the theoretical plane-wave reflection amplitudes (the blue lines) from the Zoeppritz equations in Figure 4.8a and 4.8b; no transmission loss occurs in the homogeneous layer above this reflector, and the imaged reflection coefficients are well fitted by the Zoeppritz calculations.

4.4.2 Marmousi2 model test

Figure 4.9 contains a portion of a resampled Marmousi2 model (Martin et al., 2002), that is modified by substituting solid material with non-zero S-velocity for the uppermost water layer. The grid spacings in both x- and z-directions are 5 m. One hundred sources are
Figure 4.7. Migrated (a) PP and (b) PS images using the vector-based prestack image condition; (c) and (d) are PP and PS images using normalized crosscorrelation, and using divergence and curl for P and S separation (with compensation applied). The image resolution of the vector image condition is higher because only the peak values of the source wavefield is used rather than the whole wavelet as in the normalized crosscorrelation image condition. For both image conditions, the PS images (b and d) have a higher resolution than the PP images (a and c), because the S-wave velocity is lower. See Figure 4.8 for a quantitative amplitude comparison.
Figure 4.8. Comparison of peak PP and PS image values from Figure 4.7a and 4.7b for the three reflectors in Figure 4.5a, obtained by using the vector-based (VB) image condition (the red lines), and in Figure 4.7c and 4.7d using the normalized crosscorrelation (NC) image condition (the black lines). The PP and PS image values of the uppermost reflector [in panels (a) and (b)] are also compared with values from the Zoeppritz equations [the blue lines in panels (a) and (b)]. All are displayed as a function of surface offset. The critical PP reflection angle (74°) of the uppermost reflector corresponds to surface offset of 5.2 km which is beyond the range of the data plotted here.
Figure 4.9. (a) P-wave velocity, (b) S-wave velocity and (c) density distributions used for isotropic elastic wavefield modeling. This is a portion of the Marmousi2 model.

excited on the surface of the model from x = 500 m to 4500 m with a horizontal spacing of 40 m. Receivers are located along the model surface with 5 m spacing. The time increment is 0.5 ms. A representative elastic common-source gather for a source at (x, z) = (2.5, 0.0) km is shown in Figure 4.10.
Figure 4.10. Sample elastic data simulated for the model in Figure 4.9 with source at $x = 2.5$ km and $z = 0$ km, and receivers at $z = 0$ km: (a) is the horizontal component and (b) is the vertical component of particle-velocity. The direct waves are removed.

PP and PS ADCIGs are generated by migration of all 100 common-source gathers using the P and S vector decomposition algorithm and the vector-based prestack image condition discussed above. The smoothed parameter models used for migration are shown in Figure 4.11. Every grid point in the migrated image for each shot corresponds to an incident angle, which is calculated from the P-wave Poynting vectors of the source and receiver wavefields (equation 4.19), and then image amplitudes are sorted into angle-domain common-image gathers. Consider three ADCIGs at $x = 1.5$, 3.0 and 4.0 km for both PP (Figure 4.12a) and PS (Figure 4.12b) reflections. Figure 4.13a and 4.13b are the stacked PP and PS images, which are stacked over incident angles of $-50^\circ$ to $50^\circ$ of the ADCIGs. As expected, the PS image has resolution about twice that of the PP image. The PS image doesn’t have the polarity reversal problem across the source position as shown and explained in Figure 4.7; thus the PS image will always be enhanced by stacking over sources.
Figure 4.11. Smoothed (a) P-wave velocity, (b) S-wave velocity and (c) density distributions used for the elastic prestack RTM results in Figures 4.12 and 4.13.
Figure 4.12. Representative (a) PP and (b) PS ADCIGs at three horizontal positions of \( x = 1.5, 3.0 \) and 4.0 km. The ADCIGs are flat over their respective angle apertures of \(-50^\circ\) to \(+50^\circ\). The ADCIGs are filtered along the vertical direction using a Hamming window (0.006, 0.018, 0.040, 0.050 cycles/meter).

4.5 Discussion

The vector-based prestack image condition is implemented by extending the acoustic excitation-amplitude image condition (Nguyen and McMechan, 2013) to elastic, but there are several important differences. First, the signed magnitudes of the elastic particle-velocity vectors are used instead of the pressure amplitudes. Second, for acoustic RTM using the excitation-amplitude image condition, only the maximum amplitude map and corresponding image time map need to be stored during the source wavefield extrapolation, while in the vector-based prestack image condition for elastic RTM, in addition to the image time and amplitude maps, both the P-wave particle-velocity vectors and P-wave stress component
Figure 4.13. Stacked (a) PP and (b) PS images with incident angles from $-50^\circ$ to $50^\circ$ with the vector-based prestack image condition. Direct waves were removed from the input common-source gathers before migration. Note the higher image resolution in (b) than in (a). Compare with Figure 4.9.

at the time of the maximum magnitude of P-wave particle-velocity vectors in the source wavefield over all times, at each grid point, are required for calculating the Poynting vectors to build the final image. Third, because the magnitudes of particle-velocity vectors are always positive, to get accurate reflectivity information, the signs of the reflections (both PP and PS) need to be determined and multiplied by the absolute value of reflectivity in the image condition (equations 4.22 and 4.23). Our reflection sign calculation assumes that the shape of the wavelet doesn’t change (except for the sign), which means the phase change caused by postcritical reflection, or where there is interference between a reflection and a
head wave, cannot yet be handled correctly. However, the absolute values of the reflection coefficients are still correct.

The output of the vector-based prestack image condition is an image of PP or PS reflectivity information; to increase the accuracy of the reflectivity, compensations for transmission loss and attenuation loss (e.g., Deng and McMechan, 2007, 2008) are also necessary, but are beyond the scope of the present paper. The generated PP and PS ADCIGs from the vector-based prestack image condition can be further input to prestack inversions to estimate $v_p$, $v_s$ and density (Deng and McMechan, 2008). This is also left for future work.

The theory of the vector-based prestack image condition is not limited to the excitation-amplitude image condition. With the same scheme to obtain PP/PS reflection signs from propagation directions and polarizations, a vector-based crosscorrelation or normalized crosscorrelation image condition can be formulated by saving the P-wave particle-velocity and stress $\tau_p$ components (for calculating Poynting vectors) at all time steps during the source wavefield extrapolation, and crosscorrelating the PP or PS magnitudes and multiplying by the signs at each grid point for all time steps during the reverse-time receiver wavefield extrapolation. However, the CPU expense is higher than for the excitation-amplitude image condition; a detailed comparison between the crosscorrelation and excitation-amplitude image conditions can be found in Nguyen and McMechan (2013) for the acoustic solution, and in Nguyen and McMechan (2015) for the elastic solution.

Besides the PP and converted PS images generated using particle-velocity components, stress images, as well as ADCIGs, can also be constructed; the decomposed P- and S-wave stress vectors are used instead of particle velocities, and the calculation for reflection signs should be similar to the procedure discussed above. The stress image will provide additional information for image interpretation and inversions. Note also that the ratio of stress to particle-velocity is the impedance (Helbig, 1983; Hildebrand and McMechan, 1994).

All the methodology and synthetic tests shown above are 2D, but can be extended to 3D, in which the P-wave stress $\tau_p$ remains a scalar, and following equation 4.6, is calculated using
three particle-velocity components. The reflection signs (both PP and PS) are calculated in a plane containing the incident P-wave and the reflected P- (and S-) waves for each image point at its image time. Magnitudes of the three-component particle-velocity vector are used in the deconvolutional image condition. Extensions to include attenuation and anisotropy also seem possible. The vector wavefield decomposition method for isotropic viscoelastic wavefields is also possible (Wang and McMechan, 2015b). An anisotropic vector wavefield can be decomposed into P and S vector components by solving the Christoffel equation (Zhang and McMechan, 2010; Cheng and Fomel, 2014); the relation between propagation direction and polarization needs to be explicit for the anisotropy symmetry used. The vector-based prestack image condition for anisotropic RTM needs to be further investigated.

Another related issue is multipathing. Multipathing refers to a single reflection point being illuminated by two or more waves traveling different paths between a single source and a reflection point, or between a reflection point and a receiver, with correspondingly different travel times and incident/reflection angles. The crosscorrelation image condition implicitly uses all multipaths at the expense of increasing cost, cross-talk artifacts and background noise. In this paper we keep only the reflection associated with the largest incident magnitude at each image point; this is much cheaper than crosscorrelation, but doesn’t handle multipathing. This limitation has been overcome by keeping the second and/or subsequent largest incident magnitudes and the corresponding traveltimes from different wavelets during the source wavefield extrapolation; see the MEXIC algorithm from Jin et al. (2015), the sparse crosscorrelation algorithm of Nguyen and McMechan (2015), and the multiple time path algorithm of Hauser et al. (2008).

4.6 Conclusions

A new workflow (including P and S vector decomposition and a vector image condition) is established for 2D isotropic elastic RTM using multicomponent data. P- and S-waves are
decomposed in the vector domain during both source and receiver wavefield extrapolations. Propagation directions for P- and S-waves are efficiently calculated using Poynting vectors with the decomposed P- and S-wave particle-velocity and stress vector components as input. The PP and PS reflection signs at each image point are determined using the relation between the propagation and particle-velocity directions of the source and receiver wavefields. The image condition is built in the context of the excitation-amplitude image condition using the maximum magnitude of particle-velocity vectors, the stress, and the PP/PS reflection sign at each grid point; this is a deconvolution image condition that is efficient, robust and capable of obtaining angle-dependent reflection coefficients directly from the prestack RTM. The decomposed Poynting vectors also provide the incident and reflection angle information, thus simplifying the process of obtaining PP and PS ADCIGs. Synthetic tests show acceptable results for the migrated images and ADCIGs.

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CHAPTER 5
UP/DOWN AND P/S DECOMPOSITION OF ELASTIC WAVEFIELDS
USING COMPLEX SEISMIC TRACES WITH APPLICATIONS TO
CALCULATING POYNTING VECTORS AND ADCIGS FROM
REVERSE-TIME MIGRATIONS*

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5.1 Abstract

Separations of up- and down-going and of P- and S-waves are often a part of processing of multi-component recorded data and propagating wavefields. Most previous methods for separating up/down propagating wavefields are expensive because of the requirement to save time steps to perform Fourier transforms over time. An alternate approach to separation of up-and down-going waves, based on extrapolation of complex data traces is extended from acoustic to elastic, and combined with P- and S-wave decomposition by decoupled propagation. This preserves all the information in the original data and eliminates the need for a Fourier transform over time, thereby significantly reducing the storage cost and improving computational efficiency. Wavefield decomposition is applied to both synthetic elastic VSP data and propagating wavefield snapshots. Poynting vectors obtained from the particle-velocity and stress fields after P/S and up/down decompositions are much more accurate than those without, because interference between the corresponding wavefronts is significantly reduced. Elastic reverse-time migration with both P/S and up/down decompositions shows significant improvement compared with those without decompositions, when applied to elastic data from a portion of the Marmousi2 model.

5.2 Introduction

Multi-component seismic data contain more information about the subsurface than single-component data, but it is also more difficult to process, as both P/S wave modes coexist, and waves with different propagation directions may interfere. Wavefield decomposition improves understanding and analysis of elastic multi-component wavefields. Wavefield decomposition can be based on propagation direction (e.g., up/down separation), which can be applied to both acoustic and elastic wavefields, and P/S wave mode decomposition of elastic wavefields. Both up/down and P/S decompositions can be in seismograms, which are recorded traces as a function of time, or in space-domain wavefield snapshots, at fixed-times.
5.2.1 Up/down separation

Up- and down-going wave separation in seismograms is widely used in processing ocean-bottom seismometer (OBS) data and vertical seismic profile (VSP) data. Because of the differences in acquisition geometry, the up/down separation algorithms for OBS and VSP data are different. OBS data are usually contaminated by water-layer multiple reverberations (Weglein, 1999), but can be separated into up-going waves (primaries) and down-going waves (multiples) by combining the pressure and the vertical geophone particle velocity components (White, 1965). The separated multiples can also be used in imaging of OBS data (Dash et al., 2009). VSP data recorders are distributed vertically in a borehole and thus can separate data into up-going and down-going parts in the \( \tau-p \) domain (Hu and McMechan, 1987) or the \( \omega-k \) domain (Wenschel, 1976; Hu and McMechan, 1987). Up/down separation of propagating wavefields also has significant value in removing low-frequency noise during reverse time migration (RTM) (Liu et al., 2011), and also for separating the contribution of forward and backward-scattering when computing the velocity (modulus) gradients during full waveform inversion (Tang et al., 2013; Wang et al., 2013).

In the method of Liu et al. (2011), up/down separation is implicit in a decomposed crosscorrelation image condition, and thus the separated up- and down-going wavefields are not available, which limits its applications. Poynting vectors are also used to separate the wavefields according to propagation directions (Chen and He, 2014), and are not confined to the imaging process. Poynting vectors may give inaccurate propagation directions when two or more wavefronts interfere. Local plane wave decomposition can also be used for up/down wavefield separation and for estimating propagation angles (Xie and Wu, 2002; Tang et al., 2013). Plane wave decomposition is usually performed in the frequency-wavenumber domain, which needs 4D spatio-temporal Fourier transforms of both source and receiver wavefields, which is very expensive.
An alternative approach for determining propagation directions (and hence separating up- and down-going waves), based on one-way propagation of complex traces is described by Zhang et al. (2007) for acoustic waves, and applied by Tang and McMechan (2016) to calculate multi-direction slowness vectors. A related application, based on Havlicek et al. (1998) shown by Zhang and McMechan (2011) uses Hilbert transforms to calculate instantaneous wavenumbers in \( x-z \) space to define reflector normals; another by Jin et al. (2014) calculates wave propagation directions from phase gradients. The complex trace approach is the one we use below.

### 5.2.2 P/S + up/down decompositions

All of the above studies use only acoustic data. The main new result of the present paper is the extension to elastic wave propagation. For data from elastic media, besides the up/down separation, it is essential to decompose P- and S-waves as parts of seismic data processing, such as RTM, angle-domain common image gather (ADCI G) extraction, and migration velocity analysis. P/S separation can be realized by using divergence and curl operators (Sun, 1999; Sun and McMechan, 2001) for isotropic media, or divergence-like and curl-like operators (Yan and Sava, 2008a) for anisotropic media. However, those operators alter the phase and amplitude of the original wavefield. On the other hand, P/S vector decomposition (Zhang and McMechan, 2010), which preserves all the components of P- and S-wave, has shown its superiority in terms of accuracy of both phase and amplitude in the context of data for isotropic elastic media (Wang et al., 2015). P/S vector decomposition algorithms are available for both isotropic (Ma and Zhu, 2003; Zhang et al., 2007; Xiao and Leaney, 2010; Wang et al., 2015) and anisotropic (Zhang and McMechan, 2010) media. In this paper, we combine the P/S and up/down decompositions.

In the following sections, we first illustrate the methodology of up/down wavefield separation using complex traces for multi-component data, and then combine it with P- and
Figure 5.1. Acoustic or elastic wavefields represented as slices through a $t$-$x$-$z$ volume. VSP data are represented with a $t$-$z$ slice (yellow) at a constant $x$ of the volume; an $x$-$z$ wavefield snapshot (green) is an $x$-$z$ slice at a constant $t$.

S-wave decomposition during elastic wave propagation. Examples include up/down separation of elastic VSP data and of propagating wavefield snapshots, decomposition of Poynting vectors of multiple overlapping up- and downgoing P- and S-wave. Finally, elastic RTMs are performed on a part of Marmousi2 model to generate PP and PS ADCIGs based on P/S and up/down wavefield decompositions and decomposed Poynting vectors. A comparison is made with the RTM results without decompositions and with only P/S decomposition. All the examples are in 2D, but the algorithm can be extended to 3D.

5.3 Methodology

5.3.1 Previous up/down separations ($\omega$-$k$ domain separation)

Up/down separation algorithms for elastic wavefields are similar to those for acoustic data, and are mostly based on the sign of the apparent propagation velocity $v_z$, or slowness
\( p_z \) (= \( v_z^{-1} \)) along the depth \( z \) axis. Thus the input is the \((t, z)\) slice (a VSP) at any \( x \) of a \( t-x-z \) volume, or a series of (fixed-time) snapshots on a 2D \((x, z)\) space plane (Figure 5.1). For both acoustic and elastic wavefields, \( v_z \) can be calculated using the dispersion relation produced by the 2D Fourier transform, of the input \( t-z \) slice, to \( \omega-k_z \), which gives

\[
v_z = \frac{\omega}{k_z},
\]

(5.1)

or, in term of the apparent vertical slowness,

\[
p_z = \frac{k_z}{\omega},
\]

(5.2)

where \( \omega \) is the temporal frequency and \( k_z \) is the apparent vertical wavenumber. A straightforward way to separate up- and down-going waves in VSP data (a \( t-z \) slice at fixed \( x \)), or in a wavefield snapshot (an \( x-z \) slice at fixed \( t \)) is via the 2D Fourier transform, of the \( t-z \) slice at each fixed \( x \) of the \( t-x-z \) volume, into \( \omega-k_z \) (Hu and McMechan, 1987) and using the criteria

\[
S_{D}(\omega, k_z) = \begin{cases} S(\omega, k_z), & \text{if } \omega/k_z \geq 0 \\ 0, & \text{if } \omega/k_z < 0 \end{cases}
\]

(5.3)

and

\[
S_{U}(\omega, k_z) = \begin{cases} S(\omega, k_z), & \text{if } \omega/k_z < 0 \\ 0, & \text{if } \omega/k_z \geq 0 \end{cases}
\]

(5.4)

where \( S(\omega, k_z) \) is the 2D Fourier transform of the acoustic, or a component of the elastic, wavefield (spatial dimensions other than \( z \) are omitted for simplicity), and \( \omega \) is the angular frequency. \( S_U \) and \( S_D \) are the separated up-going and down-going wavefields respectively, in the \( \omega-k_z \) domain, which are then transformed back to \( t-z \). This procedure need to be applied to each component individually of the elastic wavefield for up/down separation.

This method is applicable for separating both acoustic and elastic VSP data, where the time sequences are in the 2D \((t-z)\) recorded data. However, for separation of the propagating wavefield snapshots in \( x-z \), this becomes expensive because the time sequence must be
complete to perform the 2D Fourier transforms over time, which means that a 3D \((t-x-z)\) volume needs to be stored for time domain extrapolations, thus, making it costly. This can be simplified, with no loss of accuracy, and with a significant reduction in cost, by using complex traces.

### 5.3.2 Up/down separation using complex traces

Equations 5.3 and 5.4 require knowing \(\omega\) before up/down separation, but calculating \(\omega\) involves a temporal Fourier transform, which is expensive. However, note that only the sign of \(\omega\) is needed for the separation criteria in equations 5.3 and 5.4; thus, if the sign of \(\omega\) can be set to be "+" without changing the wavefield, then the temporal Fourier transform is no longer needed, and the process of separation into down-going waves \((S_D)\) and up-going waves \((S_U)\) can be performed in the \(t-k_z\) domain using

\[
S_D(t, k_z) = \begin{cases} 
S(t, k_z), & \text{if } k_z \geq 0 \\
0, & \text{if } k_z < 0 
\end{cases} \tag{5.5}
\]

and

\[
S_U(t, k_z) = \begin{cases} 
S(t, k_z), & \text{if } k_z < 0 \\
0, & \text{if } k_z \geq 0 
\end{cases} \tag{5.6}
\]

which are more efficient than equations 5.3 and 5.4, because they require only a 1D Fourier transform over the \(z\)-axis, which is present in both VSP data and in \(x-z\) wavefield snapshots (Zhang et al., 2007; Tang and McMechan, 2016); thus, once the sign of \(\omega\) is fixed, the up/down separation can be easily performed at any desired time step.

Seismic waves are real-valued signals, whose Fourier transforms (spectra) are complex and conjugate symmetric; the negative frequency half of the spectrum is the conjugate of the positive frequency half (e.g., Maple, Jr., 1999). Gabor (1946) and Ville (1948) create the so called "analytic" signal in signal processing, to remove this spectral redundancy. The
"analytic" signal is also called the complex trace ($\tilde{d}$) in seismology, and can be constructed using the original real-valued seismic trace ($d$) as the real part, and the Hilbert transform ($\mathcal{H}$) of the original trace as the imaginary part

$$\tilde{d}(n) = d(n) + i\mathcal{H}[d(n)],$$

(5.7)

where $n$ is the index of traces. The Hilbert transform can be calculated via Fourier transforms, but special attention needs to be given to the 0 and Nyquist frequencies; see Appendix A for a specific procedure for numerical generation of discrete-time analytic signals.

To illustrate the properties of complex traces, consider a Ricker wavelet with dominant frequency of 15 Hz and 0.5 ms time interval as the input (Figure 5.2a); this is a real signal that has a symmetric amplitude spectrum in the frequency domain (the solid blue line in Figure 5.2b). Figure 5.2c and 5.2d are the Hilbert transform of Figure 5.2a and the corresponding frequency domain spectrum. Note the amplitude spectrum (the blue line) of Figure 5.2d is the same as that in Figure 5.2b, but the phase (dashed green line) is shifted by 90°. We use Figure 5.2a as the real part and Figure 5.2c as the imaginary part to construct the complex trace Figure 5.2e, after a complex FFT, the amplitude spectrum and wrapped phase are shown in Figure 5.2f; note that all the amplitude spectrum at negative frequencies are zero, and the positive frequency half is scaled by 2; the wrapped phase is the same as that of original input data (Figure 5.2b).

The complex trace (Figure 5.2e) contains the same information (in its phase and scaled amplitude) as the input wave, and more importantly, the negative frequency part is always zero (Figure 5.2f) for any real signal as input, which makes the up/down wavefield separation scheme with equations 5.5 and 5.6 applicable. The result is that the original real-valued signal (Figure 5.2a) has now become complex-valued (Figure 5.2e). In the next subsection, we show specific examples of using complex traces to separate up- and down-going waves in elastic ($t$-$z$) VSP data and wavefield ($x$-$z$) snapshots.
Figure 5.2. (a) is the input seismic trace (a Ricker wavelet) in the time domain; (b) is the Fourier transform of (a); (c) is the Hilbert transform of (a), notice that both (b) and (d) have the same symmetric amplitude spectra; only the phases are shifted by 90°. (e) is the complex trace constructed using equation 5.7 with the (a) (blue) as real part and (c) (red) as imaginary part, and (f) is the complex trace spectrum. The solid blue and dashed green curves in (b) (d) and (f) are the amplitude spectra and wrapped phases respectively. The complex trace (e) has a non-zero amplitude spectrum only for positive frequencies (f), and its amplitude is doubled compared to (b) and (d). The 0 frequency point is shifted to the middle of the spectra in Figure 5.2b, 5.2d and 5.2f for easier visualization.

5.3.3 Algorithm implementations and applications

Up/down and P/S decompositions of elastic (t-z) VSP data

VSP data are obtained by excitation of a source on the surface, with traces recorded in a borehole, as functions of depth $z$ and time $t$. Elastic VSP data $d(t, z)$ have more than one components, and each component of the original VSP signal $d(t, z)$ can be processed to a complex signal $\tilde{d}(t, z)$ for each recording depth using equation 5.7. Then the up/down separation is achieved by forward FFTs over $z$-axis to the $t-k_z$ domain and applying the criteria in equations 5.5 and 5.6, followed by inverse FFTs back to the $t-z$ domain. Each component of the VSP data is processed separately, which means the number of resultant
pairs of separated up- and down-going waves is the same as the number of components in the original data (one pair for acoustic data, two pairs for 2C elastic data, and three pairs for 3C elastic data). Both amplitude and phase of the input data are preserved in the separated up- and down-going VSP data. This VSP separation scheme is equivalent to $\omega$-$k_z$ domain separation (Hu and McMechan, 1987).

Both P- and S-waves coexist in an elastic VSP, and are not decomposed using only this process. A viable P- and S-wave decomposition for surface ($t$-$x$) gather data is proposed by Sun (1999); Sun and McMechan (2001). In Sun’s algorithm the wavefield recorded at the surface is extrapolated downward into a velocity model to an arbitrary reference datum, then it is separated into scalar P- and S-waves by divergence and curl calculations, which are then individually extrapolated upward to reconstruct the separate P- and S-waves at their originally recorded positions; this requires amplitude and phase correction as part of the divergence and curl operations. Here, following Wang et al. (2015), we employ a similar process, but use decoupled propagation instead of curl and divergence (so corrections are not needed). This can also be applied to elastic ($t$-$z$) VSP data. The procedure uses the recorded VSP data as boundary conditions to first perform a reverse-time, leftward (or rightward, depending on the 2D geometry of the source and receiver positions) reconstruction of the propagating elastic wavefield using decoupled propagation (Ma and Zhu, 2003; Xiao and Leaney, 2010; Wang et al., 2015). The reconstructed decomposed P-wave is saved at a vertical line of virtual receivers, which is located near the real line of VSP receivers. Then a forward-time, rightward (or leftward) extrapolation is performed, with the reconstructed P-wave data, from the virtual receivers. This reconstructed P wavefield is recorded again at the same times and positions as the real VSP receivers, to give the decomposed P-wave VSP data. Finally a subtraction of the reconstructed P-wave data from each component of the original VSP yields the decomposed S-wave VSP data; see Figure 5 of Wang et al. (2015) for a particle velocity example, and see Appendix B below for a derivation for the stresses.
To illustrate the separation of elastic VSP data, we design a $256 \times 256$ two-layer model (Figure 5.3) with grid increment $h = 5$ m. The source is placed at $(x, z) = (0.45, 0.00)$ km, and the receivers are placed in a borehole at $x = 0.80$ km with vertical spacing 5 m from $(0.80, 0.00)$ km to $(0.80, 1.28)$ km (the green squares in Figure 5.3). The time increment for recording is 0.5 ms. A 2D staggered-grid finite-difference formulation is used for solving the elastodynamic (Virieux, 1986; Levander, 1988) equations, both with eighth-order accuracy in space and second-order accuracy in time. Convolutional perfectly matched layer (CPML) absorbing boundary conditions (Komatitsch and Martin, 2007) with a width of 20 grid points outside the models are applied on all four grid edges to reduce unwanted edge reflections during wave propagation. The extrapolator is used in all the following elastic synthetic tests in this, and the next, section. The CPML boundary zones are not included in the plotted snapshots or images.

The observed elastic (particle velocity) VSP data (Figure 5.4a for horizontal, and 5.4b for vertical, components) contains both up-going and down-going P- and S-waves. As the source for the VSP is offset, all receivers will record non-zero amplitudes on both components. We first decompose the elastic VSP into P (Figure 5.4c and 5.4d) and S (Figure 5.4e...
and 5.4f) parts [see the procedure described in the previous section and in Wang et al. (2015)], each with their horizontal and vertical components. Then we apply the up/down separation to each component of the decomposed P- and S-waves to obtain the down-going P-wave (Figure 5.4g and 5.4h), the down-going S-wave (Figure 5.4i and 5.4j), the up-going P-wave (Figure 5.4k and 5.4l) and the up-going S-wave (Figure 5.4m and 5.4n). The P/S decomposition of VSP data is achieved at the cost of two wavefield extrapolations, but when either the up/down separation or P/S decomposition is not required, the other can be applied directly to the original VSP data, as the processes of up/down and P/S decompositions are independent.

**Up/down and P/S decompositions of \((x,z)\) elastic wavefield snapshots**

Complex traces can also be used to separate up- and down-going waves in fixed-time \((x,z)\) snapshots during wavefield propagation. We use the complex traces as boundary conditions for complex elastic wave equation

\[
\rho \frac{\partial^2 \tilde{u}}{\partial t^2} = (\lambda + 2\mu) \nabla (\nabla \cdot \tilde{u}) - \mu \nabla \times \nabla \times \tilde{u},
\]

where, \(\lambda\) and \(\mu\) are Lamé parameters, and \(\rho\) is density. Here the extrapolated elastic wavefield \(\tilde{u}\) is complex wavefield.

Because the isotropic elastic wave equation 5.8 is linear partial differential equation, and all other parameters \((c, \lambda, \mu\) and \(\rho)\) are real numbers, the complex wave equations can either be solved directly using complex wavefield extrapolation, or decomposed into two real parts, each of which can be extrapolated separately and independently using the real and imaginary parts of the complex traces. The first term on the right side of equation 5.8 propagates only P-waves; the second term propagates only S-waves; this is how the decoupled elastic propagation works.

The specific procedures to perform the up/down separation of an \((x,z)\) propagating wavefield are shown in the flowchart in Figure 5.5 and are described as follows:
Figure 5.4. (a) and (b) are horizontal and vertical particle velocity components of the observed elastic VSP data; (c) and (d) are the horizontal and vertical components of the decomposed P-wave; (e) and (f) are the horizontal and vertical components of the decomposed S-wave; (g) and (h) are the down-going P-wave; (i) and (j) are the down-going S-wave; (k) and (l) are the up-going P-wave; (m) and (n) are the up-going S-wave.

1) Use equation 5.7 to generate a complex wavelet.

2) Extrapolate the complex wavefield from the complex source using the elastic extrapolator. Two approaches are available: (a) construct and extrapolate the complex wavefield directly by allocating complex arrays and using complex-number calculations for extrapolation, or (b) extrapolate two real wavefields using the real and imaginary parts of the complex seismograms, separately. Thus for each time step, two wavefields are calculated; one corresponds to the real part, and the other corresponds to the imaginary part, of the complex wavefield. If the stress-velocity formulation (Virieux, 1986; Levander, 1988) is
Figure 5.5. Flowchart of the procedures for the up/down wavefield separation. The input complex source or seismic traces can be generated using equation 5.7. For multi-component elastic wavefields, the forward and inverse Fourier transforms are performed on each component.
used for elastic extrapolation as we do here, both particle velocity and stress/pressure components are complex, and can be separated into up- and down-going parts.

3) At any desired time step, perform complex 1D Fourier transforms of each component of the extrapolated complex wavefield over the $z$-axis, to the $k_z$ wavenumber domain, and do separate inverse Fourier transforms of the positive and negative wavenumbers back to the space domain. The wavefield of negative wavenumbers is the up-going wavefield; the wavefield of positive wavenumbers is the down-going wavefield.

The main new result in this paper is to combine elastic P- and S-wave vector decomposition and the up/down separation, and thus to further decompose an elastic wavefield into up-going P, up-going S, down-going P and down-going S wavefields. Each decomposed wavefield also preserves all its components; and thus for all four decomposed wavefields, the phase and amplitudes are correct. The implementation is similar to the procedure discussed above; the decoupled propagation (Xiao and Leaney, 2010; Wang et al., 2015) is used in the complex wavefield extrapolation, so at each time step, we have both decomposed P/S complex wavefields, and the four (up- and down-going P, and up- and down-going S) wavefields are obtained by performing step (3) on both P- and S-wavefields separately. Benefits of performing P/S plus up/down wavefield decompositions include decomposition of the Poynting vectors, and application to elastic RTMs.

The up/down separation of a propagating elastic wavefield is performed on a 256 × 256 homogeneous isotropic elastic model (Figure 5.6a and 5.6b) with grid increment $h = 5$ m, and time increment $dt = 0.5$ ms. The source is a composite (P/S) source, which generates both P- and S-wave. A snapshot of the elastic wavefield is shown in Figure 5.6a and 5.6b for horizontal and vertical components, respectively. Decoupled propagation (cf., Ma and Zhu, 2003; Xiao and Leaney, 2010; Zhang et al., 2007; Chen, 2014; Wang et al., 2015; Wang and McMechan, 2015a) is used to decompose the elastic wavefield into P- and S-waves, followed by the up/down separation. Both P/S, and up/down separations preserve
Figure 5.6. (a) and (b) are snapshots of the horizontal and vertical components of 2D elastic wave propagation in a homogeneous elastic medium with a composite (P/S) source at the center; (c) and (d) are the components of the up-going P-waves; (e) and (f) are the components of the down-going P-waves; (g) and (h) are the components of the up-going S-waves; and (i) and (j) are the components of the down-going S-waves.

the vector information of the input wavefield. The resultant decomposed P/S wavefields have both horizontal and vertical components of the up-going and down-going waves after separation (for a total of eight components) (Figure 5.6).

5.4 Poynting vector decomposition

An important application of the composite P/S decomposition and up/down wavefield separation is in calculating Poynting vectors. The Poynting vector shows flexibility and efficiency in calculating propagation directions for obtaining angle-domain common-image gathers (ADCGs) during RTM. However, a propagating wavefield usually contains superimposed waves propagating in different directions. Poynting vectors fail to give correct propagation directions where wavefronts overlap [including where incident and reflected waves are coincident as sharp velocity boundary in the migration velocity model (Wang et al., 2016)], because the Poynting vector gives only the overall energy flow direction (Červený, 2001). This problem can be substantially alleviated by up/down separation and P/S wavefield de-
composition prior to calculating the Poynting vectors. The general particle velocity and stress form of a Poynting vector \( s \) is (Červený, 2001; Dickens and Winbow, 2011)

\[
s_j = -\tau_{jk}v_k, \tag{5.9}
\]

where \( j \) and \( k \) indicate the \( x \) or \( z \) component (in 2D); \( \tau_{jk} \) is the stress tensor for an elastic wavefield; \( v_k \) is the \( k \)th component of the particle-velocity; the repeated indices imply summation. Since we use the stress–particle-velocity formulation for extrapolation, vector components of both stress and particle-velocity are already available at each time step for calculating the Poynting vectors at each time step. An instability exists when using an amplitude or phase derivative method (not equation 5.9) to calculate propagation directions at peaks or troughs of a wavelet because the direction is undefined at these points; possible solutions include a time-lapse modification (Tang et al., 2013) or optical flow (Yan and Ross, 2013; Zhang, 2014).

We use complex wavefield extrapolations to realize up- and down-going wavefield separations; however, the variables in equation 5.9 needs to be real valued, which means that the Poynting vectors have to be calculated after the up- and down-going wavefield separation. Thus, we first decompose the P- and S-wave, and then separate these into up-going and down-going parts, and finally apply equation 5.9 for each of the four parts separately to obtain their corresponding Poynting vectors.

The decomposed P- and S-wave Poynting vectors can be calculated as (Wang and McMechan, 2015a)

\[
s_j^p = -\tau^pv_j^p, \tag{5.10}
\]

for the P-wave, where \( v_j^p \) are components of decomposed P-wave particle-velocity, and \( \tau^p \) is the P-wave stress. For the S-wave

\[
s_j^s = -[\tau_{jk} - \tau^p\delta(j - k)]v_k^s, \tag{5.11}
\]
where $v_k^s$ is the $k$th component of the decomposed S-wave particle-velocity vector, and $\delta$ is the Dirac delta function. The rationality of equations 5.10-5.11 are explained in Appendix B. All the Poynting vectors are calculated after both P/S and up/down decomposition algorithms are applied.

For illustration, consider a test with four intersecting wavefronts in a homogeneous medium; two are explosive (P-waves), and two are rotational (S-waves). Horizontal and vertical components of particle velocity of an elastic, fixed-time snapshot are captured in Figure 5.7. The region of intersection of the four wavefronts (in the green boxes in Figure 5.7) is extracted, and Poynting vectors are calculated and superimposed as the black arrows in Figure 5.8a. The length of each arrow is the relative magnitude of the Poynting vector at the grid point at the tail of the arrow. The Poynting vectors are inaccurate where the wavefronts overlap, but after the P/S vector decomposition, followed by up/down separation, the wavefield and Poynting vectors are decomposed into four parts: the up-going P-wave (Figure 5.8b), the down-going P-wave (Figure 5.8c), the up-going S-wave (Figure 5.8d), and the down-going S-wave (Figure 5.8e), in which all the corresponding Poynting vectors are pointing in their correct propagation directions, normal to the wavefronts.

### 5.5 Elastic RTM with P/S and up/down decompositions

Both the up/down and P/S wavefield decompositions and the elastic Poynting vector computations are beneficial when used in elastic RTMs and for calculating PP/PS ADCIGs, as the source and receiver wavefields are then treated in a physically correct way. We can choose the desired decomposed wavefields to construct the images while avoiding crosstalk interference with other unwanted parts, which create artifacts. Below, we use a portion of a resampled Marmousi2 model (Martin et al., 2002) (the left column of Figure 5.9) to illustrate; the model is modified by substituting solid material with non-zero S-velocity for the uppermost water layer. The grid spacings in both x- and z-directions are 5 m. Two
Figure 5.7. Snapshots of horizontal and vertical particle velocities of elastic waves from two explosive and two rotational sources. The zoomed in part of the wavefront intersection part (in the green boxes) of the horizontal component with the extracted Poynting vectors, before and after P/S decomposition and up/down separation, are shown in Figure 5.8.

hundred sources are excited on the surface of the model from $x = 500$ m to 4500 m with a horizontal spacing of 20 m. A fixed array of 1001 receivers is located along the model surface from $x = 0$ m to 5000 m with 5 m spacing; all the receivers record data from all the sources. The time increment is 0.5 ms.

To illustrate the benefits of the decompositions, three elastic RTMs are performed independently using the same recorded data; the P-velocity, S-velocity, density models are slightly smoothed (right column of Figure 5.9) to apply decoupled propagation [see Wang et al. (2015) for explanations]. All the RTM images are constructed using a forward-time extrapolated source wavefield crosscorrelated with a reverse-time extrapolated receiver wavefield; the key new aspect is that the source and receiver wavefields are decomposed.
Figure 5.8. (a) Part of the horizontal component of the elastic wavefield (in color) from Figure 5.7, superimposed with Poynting vectors before wavefield separations. After the P/S vector decomposition, followed by up/down direction separation, the wavefields and their corresponding Poynting vectors are decomposed into: (b) the up-going P-wavefield, (c) the down-going P-wavefield, (d) the up-going S-wavefield and (d) the down-going S-wavefield.

5.5.1 Elastic RTM and ADCIGs without wavefield decompositions

The first elastic RTM (Figure 5.10) is performed without up/down or P/S decompositions; the image is obtained by crosscorrelating corresponding elastic (x and z) components

\[
I_{xx}(x, \theta) = \int_0^{T_{max}} S_x(x) R_x(x) dt, \quad (5.12)
\]

and

\[
I_{zz}(x, \theta) = \int_0^{T_{max}} S_z(x) R_z(x) dt, \quad (5.13)
\]
Figure 5.9. (a) P-wave velocity, (c) S-wave velocity and (e) density distributions used for isotropic elastic wavefield modeling; (b), (d) and (f) are smoothed (a), (c) and (e) respectively. This is a portion of the Marmousi2 model.

where $S$ and $R$ are the source and receiver particle velocity wavefields, and the subscripts $x$ and $z$ denote the horizontal and vertical components; the time variable $t$ in the wavefields is implicit, and thus is omitted in these and the following equations. Crosscorrelation of components is commonly used for multicomponent elastic migrations (Yan and Sava, 2008b), but are difficult to interpret physically. The angles $\theta$ for generating ADCIGs are calculated using the Poynting vectors of equation 5.9, which give only one incident angle for each image point at each image time, thus the angle for both $I_{xx}$ and $I_{zz}$ are the same. Figure 5.10 shows the stacked $I_{zz}$ (a) and $I_{xx}$ (b) images and corresponding representative ADCIGs. The $I_{zz}$ and $I_{xx}$ ADCIGs don’t give flat events, which are expected when using the correct velocity models for migration, mainly because unseparated P- and S-waves are crosscorrelated to
generate artifacts and the angles are not calculated correctly where different wavefronts overlap. The $I_{xx}$ ADCIGs also suffer from inconsistent polarities at the same image points, which result in destructive stacking.

5.5.2 Elastic RTM and ADCIGs with P/S decomposition

The second elastic RTM (Figure 5.11) has only P/S decomposition applied, and we use the vector-based crosscorrelation imaging conditions (Wang and McMechan, 2015a),

$$I_{pp}(\mathbf{x}, \theta) = \int_0^{T_{max}} \text{sgn}_{pp}(\mathbf{x}, \theta) S_p(\mathbf{x}) R_p(\mathbf{x}) dt,$$  \hspace{1cm} (5.14)
and

\[ I_{ps}(x, \theta) = \int_0^{T_{max}} \text{sgn}_{ps}(x, \theta) S_p(x) R_s(x) dt, \]  

(5.15)

where the \( S \) and \( R \) are now the magnitudes of the source and receivers particle velocity wavefield vectors, with subscripts \( p \) and \( s \) for decomposed P- and S-waves, respectively. \( \text{sgn}_{pp} \) and \( \text{sgn}_{ps} \) denote the sign of PP and PS reflections at the corresponding image point and image time, and are determined by the relative relations between the propagation and particle motion directions [see Wang and McMechan (2015a) for detailed explanations]. The angles for PP and PS ADCIGs are calculated using the P/S decomposed Poynting vectors.

Figure 5.11 shows the stacked \( I_{pp} \) (a) and \( I_{ps} \) (b) images and the corresponding representative ADCIGs. Comparing with Figure 5.10, the ADCIGs are much flatter and more coherent; the polarities of both \( I_{pp} \) and \( I_{ps} \) are consistent over incident angles. The \( I_{ps} \) has higher resolution than \( I_{pp} \) because the wavelength of the S-wave is shorter than that of the P-wave. Also, the poor coherence of the \( I_{xx} \) ADCIGs caused by P-S crosstalk in Figure 5.10b is substantially reduced after the P/S decomposition in 5.11b, and there is a corresponding improvement in the \( I_{ps} \) stack compared to the \( I_{xx} \) stack. The low-frequency, high-amplitude artifacts observed in the \( I_{xx} \) (horizontal) component image in Figure 5.10b, are now moved to the \( I_{pp} \) stacked image; from its ADCIGs, we see the artifacts come from the large angles, and is the result of subhorizontally propagating wide angle turning waves and backscatterings. A good solution to remove these artifacts is to apply the up/down separation.

### 5.5.3 Elastic RTM and ADCIGs with both P/S and up/down decompositions

The third elastic RTM is performed with both P/S and up/down decompositions. A total of 16 images may be generated by permutation of the crosscorrelated wave modes (P or S) and propagation directions (up or down); here we show only the crosscorrelation results of down-going source P-waves \( (S^d_p) \) with up-going receiver P \( (R^u_p) \) and S \( (R^u_s) \) waves

\[ I_{pp}^{du}(x, \theta) = \int_0^{T_{max}} \text{sgn}(x, \theta) S^d_p(x) R^u_p(x) dt, \]  

(5.16)
Figure 5.11. Elastic RTM images using only P/S decomposition. (a) and (b) are $I_{pp}$ and $I_{ps}$ stacked images (left) and their corresponding representative ADCIGs (right) at four horizontal locations.

and

$$I_{ps}^du(x, \theta) = \int_0^{T_{\text{max}}} \text{sgn}(x, \theta)S_p^d(x)R_s^u(x)dt, \quad (5.17)$$

where the superscripts $u$ and $d$ denote the up-going and down-going waves. Figure 5.12 shows the stacked $I_{pp}^du$ (a) and $I_{ps}^du$ (b) images and corresponding representative ADCIGs. Generating other decomposed images (e.g., $I_{pp}^{ud}$ and $I_{ps}^{ud}$) follow the same format of equations 5.16 and 5.17, which shows the flexibility of the proposed algorithm. However, Fei et al. (2014) observed artifacts in $I_{pp}^du$ from acoustic RTMs, thus the elastic $I_{pp}^du$ images are not illustrated here. See Wang et al. (2016) for additional discussions.

Both Figures 5.11 and 5.12 give clear and correct structures of the model. The overall magnitudes in the stacks in Figure 5.12 are smaller than those in Figure 5.11, because only
a portion of the P/S waves are retained after the up/down separation. The low-frequency high-amplitude artifacts are much reduced in Figure 5.12a, because only wavefields with opposite propagation directions along the depth-axis are crosscorrelated; thus imaging of turning waves and backscatterings are avoided. The stacked PS images in Figures 5.11b and 5.12b show much wider illumination than the $I_{xx}$ in Figure 5.10b. The artifacts, which are strong at depths less than 1 km, are progressively reduced (from Figure 5.10) by applying the P/S decomposition (Figure 5.11) and then the up/down separation (Figure 5.12). There are two additional features of interest that are produced automatically in the representative PS ADCIGs in Figure 5.12b; these are that the converted wave amplitudes are correctly recovered as zero at $0^\circ$ incident angle, and that the PS reflection polarities are the same to either side of $0^\circ$. Thus no polarity correction is needed prior to stacking to obtain the $I_{ps}$ image of the left panel of Figure 5.12b. These are clear advantages of the proposed algorithm.

Another popular way to remove the low-frequency artifacts is to apply Laplacian filters. Figures 5.13a and 5.13b show the Laplacian filtered PP and PS images using the stacked images in Figures 5.11a and 5.11b as input. The low-frequency artifacts in Figure 5.11a are removed, but the phase and amplitude of the filtered images (Figure 5.13a and 5.13b) are changed compared to the input images (Zhang and Sun, 2009). On the other hand, both the amplitudes and the phases of images in Figure 5.12 are scaled the same as those in (Figure 5.11).

5.6 Discussion

The up/down wavefield separation algorithm is based on complex wavefield extrapolation. Compared with traditional (real) wavefield extrapolation, the computational cost is doubled, because a complex wavefield, instead of a real wavefield, is extrapolated. The up/down separation of an x-z snapshot can be performed at any time step, but two 1D Fourier transforms are needed at each x to perform the separation, which also increases the cost. However, it is
still much cheaper than storing the wavefields at all time steps and performing 2D Fourier transforms.

The method of Liu et al. (2011) involves real, rather than complex, wavefield extrapolation and so is more efficient when applied to acoustic RTMs. Liu’s imaging condition selectively crosscorrelates source and receiver wavefield pairs with opposite propagation directions along the z-axis, but the source and receiver wavefields are not up/down separated, and thus are not available for observation and analysis by wavefield-based applications, such as Poynting vectors. Another disadvantage of Liu’s method is that the up-down and down-up images are not separable, while the algorithm in this paper can generate these two images separately for artifact analysis [see (Wang et al., 2016) for further discussion]. More im-

Figure 5.12. Elastic RTM images using both P/S and up/down decompositions. (a) and (b) are \( I_{pp} \) and \( I_{ps} \) stacked images (left) and their corresponding representative ADCIGs (right) at four horizontal locations.
Figure 5.13. PP (a) and PS (b) images after Laplacian filtering of Figure 5.11a and 5.11b, respectively.
portantly, Liu’s imaging condition is built in the acoustic context, and cannot be applied directly to the vector-based elastic RTM imaging condition, which requires a polarity calculation (Wang and McMechan, 2015a). So the choice of algorithm depends on the context and on the desired output.

The ideas that form the basis of the up/down and P/S decomposition algorithms are straightforward and are easy to implement with minimum modifications to existing migration codes. A global Fourier transform in the vertical direction is sufficient for up/down separation of snapshot data from arbitrary velocity models because the separation does not depend on velocity (equations 5.5 and 5.6).

The elastic wavefield decomposition discussed in this paper is not perfect. A limitation exists during the up/down wavefield separation for both VSP and snapshots when the values of vertical wavenumber $k_z$ become very small (even zero), which means the waves are propagating horizontally, so a wavenumber truncation at $k_z = 0$ is introduced by the up/down separation algorithm (equations 5.5 and 5.6). This discontinuity generates artifacts in the inverse Fourier transform, so we apply a slight tapering of the low-wavenumbers for both positive and negative parts of the complex wavefield after the forward spatial Fourier transform to smooth the discontinuity, but the trade-off is losing that part of the signal. Overlapped wavefronts of the same wave mode are still inseparable if both have $k_z$ components of the same sign. However, the algorithm is not limited to up/down separation; it is also applicable for separating wavefields in any direction by changing the direction of the spatial Fourier transforms (Tang and McMechan, 2016). This allows accurate separation of incident and reflected waves that are propagating more horizontally than vertically (such as turned reflections that image salt flanks) by doing the Fourier transform in the horizontal, rather than the vertical, direction. Directional binning (Tang and McMechan, 2016) can also be used to decompose the wavefield along different directions. Ideally, the up/down separation should be referenced to the local reflection normal direction, but this is expensive.
Another related limitation in the up/down separation is when aliased noise is present in the data, either in the depth direction (in which the Fourier transform is performed), or in time (in which the Hilbert transform is performed). When data are aliased there is not a clear separation between the positive and negative apparent propagation directions (because of the spectral wraparound) so the up/down separation is also aliased.

A complication in applying the proposed separation algorithm to VSP field data occurs if the receivers are unevenly spaced in a borehole. Interpolation can be applied before the FFT over the \( z \)-axis if the data are not aliased. Another option is to use the original integral definition of the Fourier transform to handle uneven trace spacings rather than an FFT. Since wavefield extrapolations for both field and synthetic data are performed on equally spaced model grids, one need only ensure that the input seismograms are correct to perform the up/down separation of the wavefield snapshots. If the VSP well is not vertical, it is still possible to separate the up/down-going and P/S waves, provided that the data are not aliased in space along the well. As the positions of VSP receivers are known, we can still perform the leftward-rightward (or rightward-leftward) extrapolations to decompose the P- and S-waves. A modification of the approach discussed above is that the complex VSP traces can be generated first, used as boundary conditions for complex wavefield extrapolations, then up- and down-going waves can be separated in snapshots (rather than in recorded VSP data) before they are separately recorded again at the original VSP receiver positions. Inconsistency in different components of VSP data will lead to an incomplete P/S decomposition, as the relations between particle motions are distorted. This is not a problem for up/down separation, because it is done separately for the individual components, but inconsistencies between the components in the input data will remain in the separated data.

In this paper, only the down-going source (P) wavefield crosscorrelated with up-going receiver (P and S) wavefield (down-up) images are demonstrated, but a total number of 16
images can be generated for target-oriented analysis (Wang et al., 2016); e.g., the imaging of a salt-flank depends mainly on turning waves, which is better focused on the up-down images.

All the tests in this paper are based on non-dispersive elastic equations. Application to dispersive (e.g., viscoelastic) equations, is also possible, because the real and imaginary parts the complex wavefield have the same amount of dispersion during extrapolation. Further investigations are in progress.

5.7 Conclusions

An up- and down-separation algorithm, based on the properties of complex traces, is illustrated and extended from acoustic to elastic wavefields, and combined with P- and S-wave decomposition by decoupled propagation to better analyze the wavefields. The up/down separation of a propagating wavefield is achieved by using complex wavefield extrapolation, which is much cheaper than separation methods via Fourier transforms over time, because the latter requires saving the wavefield at all time steps. Tests using synthetic data show very good separation results on both elastic VSP data and on propagating wavefield snapshots. Poynting vectors computed using the up/down separated wavefields are accurate for overlapped up-going and down-going P- and S-waves. Elastic RTMs on synthetic data from the Marmousi2 model data show encouraging improvements when both up/down and P/S decompositions are applied.

5.8 Acknowledgments

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5.9 Appendix A: Generating discrete-time analytic signals

Equation 5.7 is mathematically correct, but does not include the details that are required to do a reliable numerical construction of an analytic signal. To construct the complex traces, we use the procedure of Maple, Jr. (1999) for generating the discrete-time analytic signal, as follows:

1) Assume that a discrete wavelet or a seismic trace $d(n)$ at fixed $x$ position is an $N$-point time series, and $N$ is an even number, which is preferable for FFTs. Applying a forward Fourier transform using the FFT ($\mathcal{F}$) gives its $N$-point spectrum

$$X(n) = \mathcal{F}[d(n)], \ 1 \leq n \leq N \quad (5.18)$$

2) Form the $N$-point spectrum $[Y(n)]$ of the complex trace $[\tilde{d}(n)]$ with the procedure

$$Y(n) = \begin{cases} X(1), & \text{if } n = 1 \\ 2X(n), & \text{if } 2 \leq n \leq \frac{N}{2} \\ X(n), & \text{if } n = \frac{N}{2} + 1 \\ 0, & \text{if } \frac{N}{2} + 2 \leq n \leq N. \end{cases} \quad (5.19)$$

Special treatments of the 0 frequency ($n = 1$) and Nyquist frequency ($n = \frac{N}{2} + 1$) are necessary to give the correct cyclic behavior between the negative and positive frequency parts of the spectrum (Maple, Jr., 1999).

3) Perform the inverse Fourier transform to give the complex seismic trace

$$\tilde{d}(n) = \mathcal{F}^{-1}[Y(n)], \ 1 \leq n \leq N. \quad (5.20)$$

Equation 5.7 is equivalent to equations 5.18 to 5.20 only when the 0 and Nyquist frequencies on the spectrum have no significant values.
5.10 Appendix B: Calculations of Poynting vectors from Decomposed P- and S-waves

Poynting vectors of elastic wavefields are calculated using equation 5.9, with particle velocity and stresses (Červený, 2001; Dickens and Winbow, 2011). In the (2D) matrix form

\[
\begin{bmatrix}
    s_x \\
    s_z
\end{bmatrix} = -
\begin{bmatrix}
    \tau_{xx} & \tau_{xz} \\
    \tau_{xz} & \tau_{zz}
\end{bmatrix}
\begin{bmatrix}
    v_x \\
    v_z
\end{bmatrix},
\]

which is for the P/S coupled wavefield. To calculate the Poynting vectors of P- and S-waves separately, the corresponding P- and S-wave stress tensors and particle velocities are needed.

The elastodynamic equations (Virieux, 1986; Levander, 1988) are composed of the equation of motion

\[
\rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},
\]

\[
\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x},
\]

and the general Hooke’s law

\[
\tau_{xx} = M_p \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2M_s \frac{\partial u_z}{\partial z},
\]

\[
\tau_{zz} = M_p \left( \frac{\partial u_z}{\partial z} + \frac{\partial u_x}{\partial x} \right) - 2M_s \frac{\partial u_x}{\partial x},
\]

\[
\tau_{xz} = M_s \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right),
\]

or in a matrix form

\[
\mathbf{T} =
\begin{bmatrix}
    \tau_{xx} & \tau_{xz} \\
    \tau_{xz} & \tau_{zz}
\end{bmatrix} =
\begin{bmatrix}
    M_p \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2M_s \frac{\partial u_z}{\partial z} & M_s \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \\
    M_s \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) & M_p \left( \frac{\partial u_z}{\partial z} + \frac{\partial u_x}{\partial x} \right) - 2M_s \frac{\partial u_x}{\partial x}
\end{bmatrix},
\]

where the \( u_x \) and \( u_z \) are horizontal and vertical particle displacements, and the particle velocities \( v_x \) and \( v_z \) can be calculated by taking time derivatives of \( u_x \) and \( u_z \); \( M_p \) is the P-wave modulus, and \( M_s \) is the S-wave modulus. To obtain the pure P or S wave mode
equations, one can simply make $M_s = 0$ (to get the P-wave) or $M_p = 0$ (to get the S-wave) in equations 5.24 to 5.26. Then we obtain the P-wave stress tensor

$$\boldsymbol{\tau}^p = \begin{bmatrix} M_p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) & 0 \\ 0 & M_p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) \end{bmatrix}. \tag{5.28}$$

The P-wave Poynting vector can be formed as a product of P-wave stress tensor and P-wave particle velocities

$$\begin{bmatrix} s_p^x \\ s_p^z \end{bmatrix} = - \begin{bmatrix} M_p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) & 0 \\ 0 & M_p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) \end{bmatrix} \begin{bmatrix} v_p^x \\ v_p^z \end{bmatrix}. \tag{5.29}$$

Because the off-diagonal terms of the P-wave stress tensor 5.28 are zeros, and diagonal terms are equal, thus the P-wave stress tensor can be express as a scalar $\tau^p = M_p\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)$, thus equation 5.29 and equation 5.10 are equivalent.

Similarly, The S-wave stress tensor has form

$$\boldsymbol{\tau}^s = \begin{bmatrix} -2M_s\frac{\partial u_x}{\partial z} & M_s\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) \\ M_s\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) & -2M_s\frac{\partial u_x}{\partial x} \end{bmatrix}, \tag{5.30}$$

and the S-wave Poynting vector

$$\begin{bmatrix} s_s^x \\ s_s^z \end{bmatrix} = - \begin{bmatrix} -2M_s\frac{\partial u_x}{\partial z} & M_s\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) \\ M_s\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) & -2M_s\frac{\partial u_x}{\partial x} \end{bmatrix} \begin{bmatrix} v_s^x \\ v_s^z \end{bmatrix}. \tag{5.31}$$

Notice the stress tensors have the relation (compare with equation 5.27)

$$\boldsymbol{\tau} = \boldsymbol{\tau}^p + \boldsymbol{\tau}^s, \tag{5.32}$$

thus equation 5.31 and equation 5.11 are equivalent.
CHAPTER 6

ANALYSIS OF UP/DOWN DECOMPOSED ACOUSTIC RTM IMAGES

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117
6.1 Abstract

With the up/down separation of propagating wavefields using complex traces, both (forward-time extrapolated) source and (reverse-time extrapolated) receiver wavefields are decomposed into up-going and down-going parts in acoustic reverse-time migration (RTM). Together with the crosscorrelation imaging condition, four images (down-up, up-down, up-up and down-down) are generated, which facilitate the analysis of artifacts and the imaging ability of the four images. Artifacts may exist in all the decomposed images, but their positions and types are different. The causes of artifacts in different images are explained and illustrated with sketches and numerical tests. The up-down image better images subsalt turned waves than the down-up image does; thus decomposed images have advantages over a single composite image for target interpretation.

6.2 Introduction

Prestack acoustic reverse-time migration (RTM) (McMechan, 1983a; Whitmore, 1983; Kosloff and Sherwood, 1983) has the ability to image arbitrarily complex structures without dip limitations or shadow zones, but some artifacts exist in the migrated images. For example, low-frequency noise is commonly observed in RTM images when using a migration velocity model that has sharp velocity contrasts. Many methods have been proposed to address this issue, and can be generally grouped into three categories: before, after, or during application of the imaging condition.

Yoon and Marfurt (2006), after noticing that the low-frequency noise is caused by crosscorrelating waves with large open angles, propose to limit the open angle between the incident and reflected waves via calculating Poynting vectors (Červený, 2001) before applying the crosscorrelation imaging condition. Using a similar idea, Fletcher et al. (2006) added a directional damping term in the acoustic wave equation to limit the propagation directions before applying the imaging condition.
The angle-dependent information can also be edited after applying the imaging condition when angle-domain common-image gathers (ADCIGs) are obtained. An image free from low-frequency noise can be obtained by stacking only the traces at small reflection angles within the ADCIG (Zhang et al., 2010). Noise can also be removed with a (low-frequency rejection) Laplacian filter (Zhang and Sun, 2009) of the stacked image.

Liu et al. (2011) solve the low-frequency noise problem by decomposing the source and receiver wavefields into up- and down-going components and then crosscorrelating the source and receiver wavefields with opposite propagation directions in depth in the imaging condition. Thus, their final image is implicitly the sum of two crosscorrelations: the down-going source wavefield crosscorrelated with the up-going receiver wavefield (the down-up image), and the up-going source wavefield crosscorrelated with the down-going receiver wavefield (the up-down image).

In addition to the low-frequency noise, another type of artifact is observed in RTM (Fei et al., 2010, 2014). This artifact is caused by backscattering of both incident and reflected waves at a large velocity contrast, and may form false structures in the up-down image. Thus, Fei et al. (2014) and Hu and Wang (2015) propose to use only the down-up image, rather than a sum of the down-up and up-down images.

Smoothing the velocity model significantly reduces the secondary reflections that are generated during RTMs, and thus can mitigate most of the artifacts mentioned above. However, the amount of smoothing required is model dependent. Too much smoothing will, to some extent, affect the image time and amplitudes, and may generate artificial turning waves (Nguyen and McMechan, 2015); too little smoothing will retain artifacts from the imaging of the secondary reflections (Chattopadhyay and McMechan, 2008). To limit the complexity and to clearly analyze the artifacts, we use the velocity model without smoothing (i.e., with sharp velocity boundaries) for the numerical tests below.

The application of complex traces to wavefield extrapolations was first described by Zhang et al. (2007) in the context of the one-way acoustic wave equation and verified its equivalence.
with two-way wave equation with real wavefield values. Compared with the method of Liu et al. (2011) of constructing RTM images which involves a modified image condition, a major advantage of using complex traces is that complete separated up- and down-going wavefields are available for analysis. This method is applied by Tang and McMechan (2016) to calculate multi-direction slowness vectors; Hu and Wang (2015) use it in acoustic RTM and generate down-up and up-down images separately. Wang et al. (2016) extend it to elastic wavefields in conjunction with P and S wave decomposition and application to the computation of Poynting vectors.

In this paper, we use complex traces as a tool to decompose the source and receiver wavefields, and analyze the artifacts generated in the decomposed RTM images. Beyond the previous studies of low-frequency noise (Yoon and Marfurt, 2006; Zhang et al., 2010; Zhang and Sun, 2009; Liu et al., 2011), the four decomposed images allow us to better analyze other RTM artifacts (e.g., mirror images), including the contribution of internal multiples, which cannot be examined with single component images. We also find that backscattering artifacts may appear at different positions in the decomposed images, and the up-down image [which is considered to be less useful than the down-up image (Fei et al., 2014), interestingly shows better results when turning waves are involved]. The paper is organized as follows; we first briefly illustrate the methodology of separating the up- and down-going wavefields with complex traces and the decomposed imaging condition. Then, different artifacts and their causes are illustrated with sketches. Two tests using synthetic data quantitatively show the different imaging abilities of the decomposed images and the artifacts that exist in them.
6.3 Methodology

6.3.1 Up and down separation using complex traces

A complex analytic trace $\tilde{d}(n)$ is composed with the original signal $d(n)$ as the real part, and the Hilbert transform $\mathcal{H}$ of $d(n)$ as the imaginary part (Zhang et al., 2007):

$$\tilde{d}(n) = d(n) + i\mathcal{H}[d(n)].$$ (6.1)

Unlike the real trace $d(n)$, which has a symmetric amplitude spectrum in the frequency domain, $\tilde{d}(n)$ has only a positive spectrum (Gabor, 1946; Ville, 1948). This spectral property is retained in the wavefield when using time slices from the complex traces as the boundary conditions for complex wavefield extrapolations in isotropic acoustic media. This feature avoids the expensive Fourier transform over time in wavefield separation (Wang et al., 2016; Hu and Wang, 2015), and simplifies the procedure of up/down separation into:

$$S_D(t, k_z) = \begin{cases} 
S(t, k_z), & \text{if } k_z \geq 0 \\
0, & \text{if } k_z < 0
\end{cases}$$ (6.2)

and

$$S_U(t, k_z) = \begin{cases} 
S(t, k_z), & \text{if } k_z < 0 \\
0, & \text{if } k_z \geq 0
\end{cases}$$ (6.3)

where $S_U$ and $S_D$ are the separated up-going and down-going wavefields (spatial dimensions other than $z$ are omitted for simplicity) respectively, in the $t$-$k_z$ domain, which are then transformed back to $t$-$z$ via 1D inverse Fourier transforms over the $k_z$-axis. The specific procedure of up/down separation of both acoustic and elastic vertical seismic profile (VSP) data, and for obtaining propagating wavefield snapshots, can be found in Wang et al. (2016).
6.3.2 Decomposed crosscorrelation imaging condition

During the acoustic RTM, both source \([s(\mathbf{x}, t)]\) and receiver \([r(\mathbf{x}, t)]\) wavefields are decomposed into up- and down-going components;

\[
s(\mathbf{x}, t) = s_d(\mathbf{x}, t) + s_u(\mathbf{x}, t),
\]

and

\[
r(\mathbf{x}, t) = r_d(\mathbf{x}, t) + r_u(\mathbf{x}, t),
\]

where \(s_d(\mathbf{x}, t), r_d(\mathbf{x}, t), s_u(\mathbf{x}, t)\) and \(r_u(\mathbf{x}, t)\) are the separated down-going \((d)\) and up-going \((u)\), source and receiver wavefields. Liu et al. (2011) rewrite the non-source-normalized crosscorrelation imaging condition as

\[
I(\mathbf{x}) = I_{ud}(\mathbf{x}) + I_{du}(\mathbf{x}) + I_{dd}(\mathbf{x}) + I_{uu}(\mathbf{x}).
\]

where

\[
I_{ud}(\mathbf{x}) = \int_0^{T_{\text{max}}} s_u(\mathbf{x}, t)r_d(\mathbf{x}, t)dt,
\]

\[
I_{du}(\mathbf{x}) = \int_0^{T_{\text{max}}} s_d(\mathbf{x}, t)r_u(\mathbf{x}, t)dt,
\]

\[
I_{dd}(\mathbf{x}) = \int_0^{T_{\text{max}}} s_d(\mathbf{x}, t)r_d(\mathbf{x}, t)dt,
\]

and

\[
I_{uu}(\mathbf{x}) = \int_0^{T_{\text{max}}} s_u(\mathbf{x}, t)r_u(\mathbf{x}, t)dt.
\]

In \(I_{ab}\), \(a\) is the source wavefield propagation direction and \(b\) is the receiver wavefield propagation direction with \(d\) for downgoing and \(u\) for upgoing; hence the four possible combinations of decomposed images in equations 6.7-6.10. The decomposed images can be used for better understanding the artifacts generated in RTMs.
6.4 Artifact analysis

Most artifacts observed in RTM images are associated with backscattering (secondary reflections) that are produced during source and receiver wavefield extrapolations. Depending on whether the source, receiver (or both) wavefields are backscattered, the artifacts have different characteristics and appear at different positions in the migrated image. Below, we first use simple ray-path sketches for different models to qualitatively illustrate the causes, types and positions of these artifacts, and then in the following section, numerical tests are performed on synthetic data from two models to illustrate our analysis.

The most pervasive low-frequency noise in migrated images occurs at shallow depth and is caused by either the source (or receiver) wavefield being backscattered at a strong shallow reflector, and crosscorrelated with the receiver (or source) wavefield, and usually is a combination of both. For example, when imaging a single-reflector model, low-frequency noise is observed in $I_{dd}$ (Figure 6.1a) and $I_{uu}$ (Figure 6.1b) above the reflector. The noise in $I_{dd}$ is caused by the (down-going) source wavefield crosscorrelation with the (down-going) backscattered receiver wavefield. The noise in $I_{uu}$ is caused by backscattered (up-going) source wavefield crosscorrelation with (up-going) receiver wavefield. Thus, all the grid points along the wavepath from the source and receiver satisfy the image condition and thus contribute to generating the noise. Note that these artifacts occur only when the migration velocity model contains sharp reflectors that produce reflections during the source and receiver wavefield extrapolations in the RTM. A similar situation arises with transmitted (turned) waves, where source and receiver wavefields overlap with the image condition satisfied everywhere (Chang and McMechan, 1986).

The crosscorrelation of wavefields with opposite propagation direction over depth ($I_{ud}$ and $I_{du}$) do not suffer from the low-frequency noise. Thus Liu et al. (2011) propose an imaging condition that implicitly produces a sum of $I_{ud}$ and $I_{du}$; however, a different type of backscattering-related artifacts may exist in $I_{ud}$ or $I_{du}$. Consider the velocity model with two
reflectors (Figure 6.1c and 6.1d). The down-up image \( I_{du} \) gives a clear image (Figure 6.1c), but in the up-down image \( I_{ud} \), the reflection at a second reflector is backscattered and becomes imaged above the first reflector, which forms a false structure (Figure 6.1d). This occurs because both source and receiver waves are backscattered, and the imaging condition is satisfied at locations where the backscatterings have the same combined travel time as at their correct reflector position. In the next section we quantitatively compare the image amplitudes of the \( I_{du} \) and \( I_{ud} \) from this model and also analyze contributions, of internal multiples, to the decomposed images.

Similar artifacts to those in Figure 6.1d are observed by Fei et al. (2014) from a test on data from a syncline model (Figure 6.1e and 6.1f) with a large positive velocity gradient in the overburden (or caused by model smoothing); the image \( I_{ud} \) may contain artifact events (Figure 6.1f) because of turning waves caused by the velocity gradients. Tests on synthetic data for this model can also be found in Hu and Wang (2015). These turning wave artifacts require a positive velocity gradient in the migration velocity model; the shape of the reflector does not have to be a syncline.

In some models, the illumination is limited, and reflections from certain interfaces (e.g., salt-bottoms) can only be produced by turning waves. The turning waves can be clearly imaged in \( I_{ud} \) (Figure 6.1g). \( I_{du} \), on the other hand, suffers from similar backscattering artifacts (Figure 6.1h) as in Figure 6.1d and 6.1f, but the artifacts are imaged below the correct interface locations, rather than above it. In summary, Figure 6.1 (c, e and g) shows the correctly located images, and Figure 6.1 (d, f and h) shows (misplaced) artifact images associated with the same three reflections.

### 6.5 Tests on synthetic data

In this section, numerical tests are performed on data for two models to illustrate the analysis above. The first tests use data produced by a two-reflector model similar to the
Figure 6.1. Sketches of rays in source (red) and receiver (blue) wavefields that generate images or artifacts in four model geometries. (a) and (b) are the $I_{dd}$ and $I_{uu}$ images of a single-reflector model that both cause low-frequency noise. (c), (e) and (g) are correct ($I_{du}$, $I_{ud}$ and $I_{ud}$) images respectively; (d), (f) and (h) are the corresponding mispositioned artifact ($I_{ud}$, $I_{ud}$ and $I_{du}$) images, respectively.
sketches in Figures 6.1c and 6.1d. The second tests are for data produced by a subsalt model similar to that in Figures 6.1g and 6.1h. A 2D staggered-grid finite-difference formulation is used for solving the acoustodynamic (Virieux, 1984) equation, with eighth-order accuracy in space and second-order accuracy in time. Convolutional perfectly matched layer (CPML) absorbing boundary conditions (Komatitsch and Martin, 2007) with a width of 20 grid points outside the models are applied on all four grid edges to reduce unwanted edge reflections to an acceptable level during wave propagation. The CPML boundary zones are not included in the plotted snapshots or images.

6.5.1 Numerical results for a two-reflector model

The first test is performed on a two-reflector acoustic model (Figure 6.2) with the grid increment $h = 15$ m. An explosive source (the red dot) is placed at $(x, z) = (2.0, 0.0)$ km, and the source is a 15 Hz Ricker wavelet with time increment $dt = 1$ ms. The receivers are placed at the surface with horizontal spacing of 15 m from $(0.0, 0.0)$ km to $(3.8, 0.0)$ km (the blue triangles in Figure 6.2). Figure 6.3a is the observed acoustic common-source gather; it contains two primary reflections (labeled R1 and R2) and one internal multiple (labeled R3). Figure 6.3b is the Hilbert transform of Figure 6.3a. The complex seismogram, with Figure 6.3a as the real part, and 6.3b as the imaginary part, is used as the boundary condition for the reverse-time receiver wavefield extrapolation in acoustic RTM.

Using the true velocity model (Figure 6.2) for the RTM, the original (undecomposed) migration image (corresponding to equation 6.6) is contaminated with low-frequency backscattered noise (Figure 6.4a). With the up/down wavefield separation, the low-frequency noise can be easily identified in Figures 6.4d ($I_{uu}$) and 6.4e ($I_{dd}$). Figure 6.4b ($I_{du}$) contains the reflector images (labeled R1 and R2) and is the same as a one-way wave-equation migration. Faint images of the correct reflector locations (labeled R1 and R2) can also be observed in the $I_{ud}$ image of Figure 6.4c. These images are generated when the down-going source
Figure 6.2. Two-reflector velocity model. The red dot is the source location; the blue triangles are every 14th receiver.

Figure 6.3. (a) is the common-source gather with the source at the center and top of the surface with the direct wave removed; (b) is the common-source gather in (a) after Hilbert transforming each trace in the t direction. (a) and (b) are the real and imaginary parts of the complex seismogram that is input to the RTM. Reflections labeled R1 and R2 are from the reflectors labeled R1 and R2 in Figure 6.2; reflection R3 is the first internal multiple between reflectors R1 and R2.
wavefield is backscattered into an up-going wavefield, and the up-going receiver wavefield is
backscattered into a down-going wavefield because of the large acoustic impedance contrasts
at the reflectors; the amplitude of the image is reduced by $R^2$, where $R$ is the reflection coef-
ficient at the reflector, because of the second reflection that occurs during the extrapolations
that are part of the migration. The artifact labeled $R2'$ in Figure 6.4c, is a mirror image of
$R2$, around $R1$. Both the reflections and artifacts in the $I_{ud}$ images in Figure 6.4c are much
weaker than the $I_{du}$ reflector images ($R1$ and $R2$) in Figure 6.4b.

The above images and artifacts are generated from primary reflections. An internal
multiple is also observed in the seismograms of the two-reflector model (labeled $R3$ in Fig-
ure 6.3a), but its relative amplitude is much smaller than those of reflections $R1$ and $R2,$
because it has been reflected three times (at the reflection points b, d and e in Figure 6.5b);
see $m0$, which is the raypath that generates these multiples. To observe the contributions
of multiples to the images, the down-up and up-down images are replotted in Figure 6.5 (a
and c) with a higher amplitude scale factor than in Figure 6.4 (b and c).

Because the wavefield cannot be perfectly reconstructed using data from only one bound-
ary, the multiples, during receiver wavefield extrapolation, may not follow the original
wavepaths from which they are generated. In the replotted down-up image (Figure 6.5a),
another artifact $M1$ can now be observed. This is produced when the source waves and re-
ceiver multiples transmit through the two reflectors and are imaged below the lower reflector;
the corresponding raypath for generating this artifact is marked as $m1$ in Figure 6.5b. This
is the mirror image artifact associated with the correct internal multiple image at location
b. Similar to the backscatterings of primary reflections in Figure 6.1d, multiples are also
backscattered during extrapolations. The multiple backscatterings may form artifacts or cor-
rect images depending on where the multiples are backscattered. When the source wavefield
and the multiples from the receiver wavefield are backscattered at a and c respectively, of
the upper reflector (Figure 6.5b), a mirror artifact of $M1$ (Figure 6.5a) is formed above the
Figure 6.4. RTM images obtained using the acoustic data in Figure 6.3b and 6.3c and the true velocity model (Figure 6.2). (a) is the crosscorrelation image without up/down separation; (b) is the down-up image; the images labeled R1 and R2 correspond to the reflectors R1 and R2 in Figure 6.2. (c) is the up-down image; the amplitudes are much smaller than in (b), and note the artifact labeled R2'; (d) is the up-up image; (e) is the down-down image. (d) and (e) contain mainly low-frequency noise associated with the paths in Figure 6.1 (a and b).
upper reflector; the corresponding raypath is labeled as m1′ in Figure 6.5d. For this model, the position of M1′ is above the upper surface of the model, and thus cannot be observed in Figure 6.5c. On the other hand, when the backscatterings occur at d and e of the lower reflector, then the raypath (m2 in Figure 6.5d) is the same as m0 (in Figure 6.5b), which is the original raypath that generated these multiples; thus the multiples are correctly imaged at M2 in Figure 6.5c. However, M2 has the same position and polarity as the primary reflection image R1 in Figure 6.5c, making them difficult to be distinguished. Both M2 and M1′, if observable, are weaker than M1 in amplitude, because the transmission coefficients in this model are much larger than the reflection coefficients.

6.5.2 Numerical results for the Sigsbee model

Another test is performed using synthetic acoustic data from the Sigsbee model (Figure 6.6) (Paffenholz et al., 2002) with grid increment \( h = 5 \) m and time increment \( dt = 0.5 \) ms; 8001 time steps are recorded for each source. As with the layered model test (above), Figure 6.7a-c are the stacked RTM images for 20 sources, evenly spaced from \((x, z)=(0.9, 0.0)\) km to \((2.5, 0.0)\) km (Figure 6.6); the data for all sources are recorded with the same fixed array of receivers from \((0.0, 0.0)\) km to \((8.0, 0.0)\) km with spacing of 5 m.

Figure 6.7a is the crosscorrelation image without up/down wavefield separation (corresponding to equation 6.6), and thus shows the expected high-amplitude, low-frequency artifacts above the water bottom and salt reflectors. Figure 6.7b and 6.7c are the down-up \( (I_{du}) \) and up-down \( (I_{ud}) \) images, respectively, which are free of the low-frequency noise. The down-up image in Figure 6.7b shows clear geological structures migrated from non-turning waves, but contains misplaced artifacts in subsalt areas (in the red oval in Figure 6.7b). See the previous section for explanations.

On the other hand, \( I_{ud} \) shows improved S/N ratio in the subsalt area (see the red oval in Figure 6.7c), because the subsalt image depends mainly on turning waves, which are
Figure 6.5. Artifacts produced by internal multiples. Down-up (a) and up-down (c) images of Figures 6.4b and 6.4c with a higher amplitude plotting scale factor; (b) and (d) are their corresponding raypath sketches to illustrate the corresponding images/artifacts. The solid raypath m0 in (b) indicates how the internal multiples are generated; the dashed lines m1, m1' and m2 are possible raypaths for imaging the multiples during RTM; M1 and M2 in (a) and (c) are the corresponding images/artifacts. M1' is not labeled in (c) because it is imaged outside the model [see panel (d)].

composed of the up-going source waves and down-going receiver waves at the salt bottom produced by the gradually increasing velocity with depth in the sediments. $I_{ud}$ suffers from reduced image amplitudes and the backscattered artifacts [compare in the blue ovals in Figure 6.7 (b and c)], which is the same type of artifacts that exist in $I_{du}$, but at different positions. Additional stacking over many more shots may partially reduce these problems, but the amplitudes of multiple artifacts in the subsalt area in $I_{du}$ are much larger than the signal in $I_{ud}$. Thus, combining $I_{ud}$ and $I_{du}$ is not advisable when the subsalt region is the
Figure 6.6. P-wave velocity of the Sigsbee model. The red dots represent every 2nd source position. The receivers are evenly distributed from (0.0) km to (8.0, 0.0) km with spacing of 5 m.

main target for investigation; the individual decomposed images contain more independent information for analysis.

6.6 Discussion

In this paper, only RTMs of surface data from 2D acoustic models are shown, but the migration algorithm is also applicable to acoustic RTMs of VSP data and to elastic RTMs of surface or VSP data. Elastic wavefields contain both P- and S-waves, and so result in a total number of 16 images when both up/down and P/S decompositions are applied to both source and receiver wavefields (Wang et al., 2016). For both acoustic and elastic media, up/down and P/S separations are easily extended to 3D, anisotropic, and viscoelastic data.

With complex traces, the cost of RTM is doubled, because a complex wavefield is extrapolated, instead of a real wavefield. In addition, the process of up/down separation requires two 1D Fourier transforms per trace, which also increases the cost.

A common problem for all up/down wavefield separation algorithms is that the horizontally or near-horizontally propagating waves are difficult to handle. When using complex traces, those waves are filtered out in the wavenumber domain to avoid discontinuity ar-
Figure 6.7. Stacked RTM images of the Sigsbee model test with 20 sources using the true velocity model; (a) is the crosscorrelation image without up/down wavefield separation; (b) is the down-up image; (c) is the up-down image. Note the subsalt area (the red oval) is better imaged in (c) than in (b), because this area depends on mainly turning waves. However, for near-offset areas (e.g. the blue ovals) the up-down image (c) suffers more from the backscattering artifacts than (b).
Artifacts in the Fourier transforms. As a consequence, those waves cannot be imaged after up/down separation. A solution is to perform multi-direction, rather than only up/down, separations (Tang and McMechan, 2016). This will generate more images for analysis, with a corresponding increase in cost.

The gradient in full waveform adjoint inversion has a similar crosscorrelation form as that in the imaging condition of RTM, thus the gradient can also be decomposed with the same procedure. However, the up-up ($I_{uu}$) and down-down ($I_{dd}$) parts, which are considered noise in migration, are more valuable in FWI, as they can be used to invert for the low-frequency components of the model parameters (Tang et al., 2013; Wang et al., 2013).

6.7 Conclusions

Based on the decomposed acoustic RTM images via up/down separation, we have analyzed the origin and characteristics of various types of artifacts, and the differing imaging abilities of the corresponding decomposed images. Backscattering artifacts may exist in all of the decomposed images, but in different positions. The up-down image has lower image amplitudes than the down-up image, but the former can better image the turning waves, which is more reliable in subsalt regions. Thus we recommend using the decomposed images, rather than the single composite one, to interpret target details.

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CHAPTER 7
CONCLUDING REMARKS

7.1 Summary

We analyze and compare two affordable methods for isotropic elastic decomposition in the vector-domain (selective attenuation and decoupled propagation). Both are performed during extrapolation. Synthetic tests show that both methods give accurate decomposition results for homogeneous and smoothly varying isotropic models. The decoupled propagation is better in terms of accuracy, speed, memory requirement and numerical stability. Decomposition of the observed $x$-$t$ data into P and S vector seismograms can be achieved by downward, followed by upward elastic wavefield extrapolations through a homogeneous model with either of the wavefield-decomposition algorithms, and reconstruction of the decomposed seismograms.

We extend the vector-domain P- and S-wave decoupled vector decomposition algorithm from isotropic to viscoelastic wavefields. Both amplitude and phase information are preserved. Synthetic tests show that the proposed decoupled viscoelastic vector decomposition method gives accurate decomposition results for data from inhomogeneous, isotropic, viscoelastic models.

We establish a new workflow (including P and S vector decomposition and a vector image condition) for 2D isotropic elastic RTM using multicomponent data. P- and S-waves are decomposed in the vector domain during both source and receiver wavefield extrapolations. Propagation directions for P- and S-waves are efficiently calculated using Poynting vectors with the decomposed P- and S-wave particle-velocity and stress vector components as input. The PP and PS reflection signs at each image point are determined using the relation between
the propagation and particle-velocity directions of the source and receiver wavefields. The image condition is built in the context of the excitation-amplitude image condition using the maximum magnitude of particle-velocity vectors, the stress, and the PP/PS reflection sign at each grid point; this is a deconvolution image condition that is efficient, robust and capable of obtaining angle-dependent reflection coefficients directly from the prestack RTM. The decomposed Poynting vectors also provide the incident and reflection angle information, thus simplifying the process of obtaining PP and PS ADCIGs. Synthetic tests show acceptable results for the migrated images and ADCIGs.

An up- and down-separation algorithm, based on the properties of complex traces, is illustrated and extended from acoustic to elastic wavefields, and combined with P- and S-wave decomposition by decoupled propagation to better analyze the wavefields. The up/down separation of a propagating wavefield is achieved by using complex wavefield extrapolation, which is much cheaper than separation methods via Fourier transforms over time, because the latter requires saving the wavefield at all time steps. Tests using synthetic data show very good separation results on both elastic VSP data and on propagating wavefield snapshots. Poynting vectors computed using the up/down separated wavefields are accurate for overlapped up-going and down-going P- and S-waves. Elastic RTMs on synthetic data from the Marmousi2 model data show encouraging improvements when both up/down and P/S decompositions are applied.

We analyze the origins and characteristics of various types of artifacts, and the differing imaging abilities of the corresponding decomposed images. Backscattering artifacts may exist in all of the decomposed images, but in different positions. The up-down image has lower image amplitudes than the down-up image, but the former can better image the turning waves, which is more reliable in subsalt regions. Thus we recommend using the decomposed images, rather than the single composite one, to interpret target details.
7.2 Future work

Future work includes extension of vector-based elastic RTM from 2D to 3D, PS decomposition of multi-component seismograms with topography, velocity inversion based on migrated images and ADCIGs, and complete analysis of up/down P/S decomposed elastic images.
REFERENCES


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