The Strategic Value of High-Cost Customers

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Many firms today manage their existing customers differentially based on profit potential, providing fewer incentives to less profitable customers and firing unprofitable customers. Although researchers and industry experts advocate this practice, results have been mixed. We examine this practice explicitly accounting for competition and find that some conventional prescriptions may not always hold. We analyze a setting where customers differ in their cost to serve. We find that when a firm can discriminate among its customers but the rival cannot, customer base composition influences the rival’s poaching behavior. Consequently, even though a low-cost customer is more profitable when viewed in isolation, a high-cost customer may be strategically more valuable by discouraging poaching. Therefore, contrary to conventional advice, it can be profitable for a firm to retain unprofitable customers. Moreover, some customers may become more valuable to retain and receive better incentives when they are less profitable. We further show that, in competitive settings, traditional customer lifetime value metrics may lead to poor retention decisions because they do not account for the competitive externality that actions toward some customers impose on the cash flows from other customers. Our results suggest that firms may need to evolve from a segmentation mindset, which views each customer in isolation, to a customer portfolio mindset, which recognizes that the value of different customers is interlinked.

Keywords: competitive strategy; customer lifetime value; customer profitability; customer relationship management; dynamic competition; price discrimination; game theory

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Some of my clients now tell me they’re sorry they listened to consultants five years ago when we told them to get rid of unprofitable customers. …Today, they’re saying, “To heck with that, we want to serve every customer we can.”
—Vice president at a leading management consulting firm (Strategy + Business 2002)

1. Introduction

More and more firms are recognizing and managing their customers as assets. Firms track their interactions with individual customers, and assign both the revenues and costs of serving each of them. Firms are then able to calculate the lifetime profits of each customer and manage their existing customers differentially, often offering a better price, service, or other incentives to more profitable customers, while limiting their spending on less profitable ones. For example, FedEx Corporation classifies its business customers as “good,” “bad,” and “ugly” based on their profit potential and provides better service to more profitable customers while charging higher prices to less profitable ones (Selden and Colvin 2003). Fidelity Investments routes calls from less profitable customers to longer queues and charges them for services that are free to more profitable customers (Selden and Colvin 2003). At AT&T Wireless, customer retention incentives such as cell phone subsidies or free airtime are based on customer profitability (Richtel 2006).

Many researchers and industry experts support this practice. Customer base analysis often reveals that some customers contribute to a large percentage of a firm’s profits, although a substantial proportion are unprofitable and destroy firm value. It is argued, therefore, that if a firm treats all its customers equally, it not only wastes resources on unprofitable customers, but also effectively underserves the profitable ones and risks losing them (e.g., Rust et al. 2000, Niraj et al. 2001, Selden and Colvin 2003, Venkatesan and Kumar 2004, Gupta and Lehmann 2005).

Firms are also advised to make their unprofitable customers profitable, for instance, by charging them a higher price. It is believed that doing so can only benefit the firm, because if they stay with the firm, they become profitable, whereas if they leave, the firm’s profitability still improves. Moreover, if
some unprofitable customers cannot be made profitable, it is recommended that they be "fired" (e.g., Rust et al. 2000, Selden and Colvin 2003, Gupta and Lehmann 2005, Mittal et al. 2008). For instance, Sprint Wireless issued disconnection letters to some of its customers because they called customer support too often (Srivastava 2007). The logic here is quite compelling: if a firm only has profitable customers, then this should boost its profitability.

However, despite the potential benefits and the compelling rationale, firms that have adopted this practice have not always realized the desired outcomes. For instance, the U.S. retail banking industry, one of the largest adopters of this practice, has been circumspect about whether the practice has boosted profitability (e.g., Stoneman 1999, Hallenborg 2000). In a 2005 survey, only a third of the banks reported being satisfied with their results (SAS Institute and Carlson Marketing Group 2005). More interestingly, as reflected in the opening quote of this paper, some banks have even become skeptical of alienating unprofitable customers (Strategy + Business 2002).

One possible reason why actual results may have diverged from expected outcomes could be that, hitherto, researchers and industry experts in this area have mostly studied firms in isolation. As a result, although their recommendations are intuitively sound and well-founded under many conditions, they do not account for competitive interactions in the marketplace. Indeed, some researchers have identified this as an important gap in the current perspective (e.g., Boulding et al. 2005, Gupta et al. 2006, Zeithaml et al. 2006, Villaneuva and Hanssens 2007).

Our objective in this paper is to conduct a model-based investigation of this practice explicitly accounting for competition, and to compare our results with prescriptions from research and popular press that do not consider competition explicitly. In particular, we are interested in studying firms' forward-looking retention strategies when managing customers as assets. To focus on this issue, we examine a two-period setting where each firm has an existing customer base to begin with. Customers differ in their cost to serve and a customer’s cost is known only to her current firm. As we explain later below, these features distinguish our model from much of the prior work.

We find that when a firm is better informed about its existing customers than a rival firm, customer base composition influences the rival’s poaching behavior, and thereby affects the profitability of the overall customer base. As a result, even though a low-cost customer is more profitable when viewed in isolation, a high-cost customer may be strategically more valuable by making poaching less attractive for the rival. For that reason, a firm may find it profitable under certain conditions to “poison” its customer base by having an additional high-cost customer rather than an additional low-cost customer. It may even be optimal to retain unprofitable customers. To our knowledge, this strategy of poisoning one’s own customer base for profit gain in a competitive context has not been previously noted in the customer relationship management (CRM) literature and is qualitatively different from conventional prescriptions. Table 1 summarizes our findings in this regard.

### Table 1 Summary of Findings

<table>
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<tr>
<th>Implications for measuring customer value</th>
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<td>1. Traditional metrics may systematically underestimate the value of less profitable customers and overestimate the value of more profitable ones.</td>
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<tr>
<td>2. Traditional metrics may have to be modified to account for the competitive externality that actions toward some customers may impose on the cash flows from other customers.</td>
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1. A firm may optimally retain unprofitable customers without “firing” them or raising prices.

1.1. Relationship to Past Literature

1.1.1. Dynamic Competitive Price Discrimination

Prior research has mostly examined settings where a firm discriminates between its existing customers and the rival’s customers or new customers (e.g., Chen 1997, Villas-Boas 1999, Fudenberg and Tirole 2000, Pazgal and Soberman 2008, Zhang 2011). The basis for discrimination in these models is that, on an average, existing customers have a stronger preference for the firm compared to other customers. Our work differs from this literature in three ways. First, we are primarily interested in how a firm discriminates among its existing customers, rather than between existing and new customers. Second, as discussed in Shin and Sudhir (2010) and Subramanian et al. (2008), models with only preference-based discrimination do not capture the notion that firms should treat their more profitable customers better. In contrast, discrimination based on cost to serve does, providing a more appropriate setting to evaluate conventional prescriptions for managing

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1 For example, Rust et al. (2000, p. 188) note, “Many customers are too costly to do business with and have little potential to become profitable, even in the long term. . . . In most cases it is desirable for a firm to alienate or even ‘fire’ at least some of its customers.”

2 We thank an anonymous reviewer for suggesting this positioning for our work.
customers based on their profit potential. Last, prior research has not examined how a firm may differentially retain its existing customers taking into account future competition for these customers, and hence cannot investigate how a firm’s customer retention strategy affects competitive intensity in the marketplace. In this context, we show that a firm can increase its profits by strategically poisoning its own customer base to discourage the rival’s poaching behavior.

A different competition-dampening effect arises in Villas-Boas (1999): forward-looking firms compete less intensely than myopic firms to acquire new customers because acquiring a larger customer base in one period precommits the firm to compete more aggressively for new customers in future periods, thereby lowering future profits. In our setting, it is the composition of the customer base (not its size) that gives rise to the competition-dampening effect.

Musalem and Joshi (2009) study competitive CRM when customers’ characteristics are known to all firms. Hence, customer base composition does not affect competition in their setting. Shin and Sudhir (2010) show in a two-period setting that when firms discriminate (in the second period) based on both purchase histories and purchase volume, firms may reward their own high-volume customers more than the rival’s customers. Villas-Boas (2004) finds that when a monopolist can discriminate between new and existing customers, it can lead to cycles in the price being offered to new customers. Last, Shin et al. (2012) examine a monopolist serving customers who differ in their cost to serve as well as their willingness to pay. They also consider that customers can endogenously decide their type by choosing the level of service to demand. They find that customer cost-based pricing might hurt a firm because a forward-looking customer may delay their purchase or reduce their service demand to avoid a future higher price. They also find that the monopolist may charge a high price and effectively “fire” high-cost customers who have a lower willingness to pay, including some who are profitable, since such a high price allows the monopolist to extract more surplus from those who have a higher willingness to pay.

1.1.2. Strategic Effect of Higher Costs. Higher costs may lead to higher prices and softer competition. The question, however, is whether a firm can benefit from this. Prior research has shown that a firm can benefit from raising its rival’s costs to create a cost advantage for itself (e.g., Salop and Scheffman 1983, Salinger 1988, Subramanian et al. 2013). In our case, a high-cost customer is equally costly for both firms. Researchers have also shown that, in various settings, an otherwise negative change (e.g., higher cost) may lead to less aggressive behavior from a rival, and thus may benefit the firm (e.g., Fudenberg and Tirole 1984, Bulow et al. 1985, Cabral and Villas-Boas 2005); that is, a positive strategic effect may offset the direct negative impact. In particular, Bulow et al. (1985) and Cabral and Villas-Boas (2005) show this in the context of multimarket and multiproduct competition, where there are cost or demand interactions across market. In our context, the different customer types may be viewed as different markets, and there is an interaction across markets when the rival firm cannot discriminate between these customer types. Thus, our work may be seen as an application of the mechanism identified in prior work to the CRM context.

Finally, our work is related to research on credit market competition that has shown that when lenders are asymmetrically informed about the creditworthiness of borrowers, this can soften interest rate competition, because an informed lender denies credit to “high-risk” borrowers, thus making uninformed lenders more cautious (e.g., Broecker 1990, Sharpe 1990, Rajan 1992, Dell’Aricia et al. 1999, Villas-Boas and Schmidt-Mohr 1999, Boot and Thakor 2000, Banerjee 2005). However, to the best of our knowledge, researchers have not examined how an informed lender may strategically manage the composition of its loan portfolio. When the impact of customer composition is accounted for, we show that a firm may find it profitable to retain unprofitable customers.

2. Model

We consider a market with a unit mass of customers served by two firms, A and B. Customers differ in their cost to serve because of how they buy or use a firm’s offerings. For example, Fidelity Investments found that a considerable proportion of its customers were unprofitable because they contacted representatives too frequently (Selden and Colvin 2003). Similarly, in a business-to-business context, Niraj et al. (2001), found that customers differed in their cost to serve because of differences in purchasing behavior linked to business size, decentralization of purchasing, inventory holding policy, reorder policy, number of delivery locations, and payment habits. We further note that customers who may otherwise be similar in terms of what they buy, can still differ considerably in their cost to serve because of how they buy and use the firm’s offerings (e.g., Selden and Colvin 2003, Niraj et al. 2001). Therefore, we assume that a firm can know a customer’s cost to serve only if it has actually served the customer.

We assume that there are two types of customers in the market, namely, high-cost customers and low-cost
customers, denoted by \( h \) and \( l \), respectively. Let \( c_j \) be the cost to serve a customer of type \( j \in \{ h, l \} \), such that a firm that serves this customer and charges a price \( p \) earns a profit of \( p - c_j \) per period. We have that \( c_h > c_l \). Let \( \theta_j \in (0, 1) \) be the proportion of type \( j \) customers, such that \( \theta_h + \theta_l = 1 \). Let \( \epsilon = \theta_h c_h + \theta_l c_l \) denote the average cost to serve. Since our main objective is to study how a firm manages its customer portfolio taking into account the competitive context and future ramifications, we consider that both firms have an initial base of existing customers and examine two periods of interaction, namely, periods 1 and 2. Since we will assume that the firms are symmetric in all respects, we take the starting configuration of the initial customer bases to be also symmetric, with each firm’s customer base consisting of half the customers of each type in the market.\(^4\)

### 2.1. Consumer Model

Each customer buys one unit of a product or a service in each period from one of the firms. We assume that both customer types have the same base preference for either firm’s offering. In addition, we assume that customers experience a random preference shock reflecting situational or contextual factors that vary from one buying occasion to the next. For instance, behavioral researchers have shown that various aspects of the decision environment as well as a consumer’s mood, emotions and feelings can influence consumer decisions (e.g., Schwartz and Clore 1983, Payne et al. 1993). Such factors and their influence are likely to vary across customers and over time, and are not easily observed by the firm. Specifically, for a customer \( k \) of type \( j \), in period \( T \in \{ 1, 2 \} \), let \( U_{ijT}^k \) be the utility from firm \( i \)’s offering, \( i \in \{ A, B \} \), given by

\[
U_{ijT}^k = V_i^k + \xi_{ijT}^k,
\]

where \( V_i > 0 \) is the base utility from the product or service for a type \( j \) customer, and \( \xi_{ijT}^k \) is a zero-mean shock that is independent across customers and across periods for each customer and is not observed by the firm. What will be relevant for our analysis is the distribution of \( \xi_{ijT}^k - \xi_{ijT}^l \). We assume that \( \xi_{ijT}^k - \xi_{ijT}^l \) is distributed uniformly over \([-1, 1]\) for both customer types. We assume that the customer’s net utility when she is charged a price \( p \) is given by \( U_{ijT}^k - p \).

Some clarifications about our assumptions may be in order. First, since the preference shocks are independent across periods, they do not form a basis for discrimination across customers. This enables us to study the implications that arise solely due to discrimination based on cost to serve. As noted earlier, preference-based discrimination by itself does not capture the notion that firms should treat their more profitable customers better, and thus may not provide an appropriate setting to evaluate the conventional prescriptions. Our assumption also implies that customers’ relative preference for firms is independent across periods.\(^5\) Second, since \( \xi_{ijT}^k - \xi_{ijT}^l \) has the same uniform distribution across customer types, both customer types are equally price sensitive. This enables us to study the implications that arise solely due to discrimination based on cost to serve.\(^6\)

Later, in §D.4 of the online supplemental appendix (available at http://papers.ssrn.com/abstract=2319192), we discuss the implications when high-cost customers are also more price sensitive. Last, our assumptions taken together constitute a random-utility formulation that yields a linear (in price) choice probability model. This enables us to keep the analysis over two periods tractable.\(^7\) We also note that previous research in marketing and economics has used random-utility formulations to study firm strategies (e.g., Musalem and Joshi 2009, Anderson et al. 1992).

In our main model, we assume that customers are myopic and make their purchase decisions to maximize their current period utility in each period. This enables us to study the strategic considerations that arise solely due to the competitive interaction between firms and introduce our main insights. This setting is also more tractable, allowing us to derive all our results analytically. We further note that conventional prescriptions are not explicitly conditioned on whether customers are forward looking or myopic, and are expected to hold regardless. Thus,

\(^4\)In a previous version of this paper, we also examined initial competition to acquire a customer base. We showed that owing to the symmetry between the firms, each firm acquired half the customers of each type. Therefore, in the absence of asymmetry between firms, we do not consider an asymmetric starting configuration in the current model. The details of the analysis are available from the authors upon request.

\(^5\)In §D.5 of the online supplemental appendix, we examine two situations (inertial loyalty and brand loyalty) where customers have a persistent preference for one of the firms and firms can discriminate on this basis. We find that our main insights continue to hold qualitatively.

\(^6\)By itself, discrimination based on price sensitivity also does not capture the notion that firms should treat their more profitable customers better; less price sensitive customers are more profitable and will be charged higher prices.

\(^7\)We note that random-utility models with linear choice probabilities have been used in several empirical applications. Heckman and Snyder (1997) provide a formal development for the linear choice probability model as a random-utility model. In particular, they show how it may be derived from independent shocks whose difference follows a zero-mean uniform distribution. Further, they allow for the preference shocks to not be identically distributed and note: “The assumption made in random-utility theory, that shocks to preferences are identically distributed, is an artificial convention” (Heckman and Snyder 1997, p. 143). For our purposes, we directly assume the distribution of the difference in the preference shocks to be uniform.
it can be useful and relevant to know whether they hold when customers are myopic. We recognize, however, that it is important to understand whether our results continue to hold when customers are forward looking. Therefore, in §5, we extend our analysis to forward-looking customers to verify that they indeed hold. As a note of caution, some of our results in this setting require numerical analysis as explained in §5. Therefore, they must be interpreted with care.

2.2. Firm Decisions

At the start of period 1, each firm learns the individual types of its existing customers and can tailor its price to these customers based on their type. Firms, however, do not know the types of their rival’s customers and cannot discriminate among them. Let \( p_{ij} \) denote firm \( i \)'s price to its existing customers of type \( j \), and let \( p_{ijR} \) denote its price to its rival’s customers. At the start of period 2, each firm also learns the individual types of the customers it acquired in period 1. Let \( p_{ij2} \) be firm \( i \)'s price to its existing customer of type \( j \), and let \( p_{ijR2} \) be its price to its rival’s customers. Firms set all prices simultaneously in each period.

To begin with, we assume that a firm does not keep track of customers who switched to the rival in period 1 and therefore charges the same price to all of the rival’s customers in period 2. This is consistent with what is observed in some industries; for instance, customers who switched their cable, wireless, or internet service provider can usually switch back in the future to their original service provider availing all “new customer” discounts. One reason why firms may not retain (or use) information about old customers could be due to customer privacy concerns. For instance, the Federal Trade Commission (2010) is considering regulating the information that firms retain about past customers, including the duration for which such information is retained. Thus, we initially assume that firms do not retain information about their past customers due to legal or regulatory concerns. This assumption also makes our analysis more tractable, allowing us to derive explicit solutions for the equilibrium strategies and to examine various extensions.

In firm \( i \)'s customer base, let \( S_{ijT} \) be the number of type \( j \) customers at the start of period \( T \), and let \( x_{ijT} \) be the proportion of these customers retained in period \( T \) by firm \( i \). We have

\[
S_{ij1} = \frac{1}{2} \theta_j, \quad S_{ij2} = \frac{1}{2} \theta_j [x_{ij1} + (1 - x_{ij1})],
\]

where \( i, j \in \{1, 2\} \) and \( i \neq i' \). In period \( T \), let \( \Pi_{ijT}^O \) and \( \Pi_{ijT}^R \), respectively, be the current period profits from firm \( i \)'s own \( (O) \) and rival’s \( (R) \) customer bases. Let \( \Pi_{ijT} = \Pi_{ijT}^O + \Pi_{ijT}^R \) be its current period profits across both customer bases. Let \( \pi_{ijT}^O = p_{ijT}^O - c_j \) be the current period profits from a (single) customer of type \( j \) retained by firm \( i \) in its customer base, and let \( \pi_{ijT}^R = p_{ijT}^R - c_j \) be the current period profits from a customer of type \( j \) acquired by firm \( i \) from its rival. Let \( \Pi_i = \Pi_{i1T} + \Pi_{i2T} \) be firm \( i \)'s total profits. Table 2 summarizes our model notation. (Some variables are introduced later.)

We assume that firms are forward looking, maximize their expected profits, and do not discount future profits. We assume that \( c_j, \theta_j \), and the distribution of \( \xi_{ijT} - \xi_{ijRT} \) are common knowledge. In period 2, firms must take into account the composition of each customer base when setting their prices. Since firms have information about their own customers and know the overall market composition, they can essentially infer the composition of the rival’s customer base. In practice, firms may conduct market surveys or benchmarking studies to understand the composition of the overall market.

To begin with, we assume that \( V_j > 2 + c_j \) such that the market is fully covered, and all customers are willing to pay more than their cost to serve; that is, they are all potentially profitable to serve, although unprofitable customers may arise endogenously in this setting. Later, in §D.3 of the online supplemental appendix, we examine a situation where high-cost customers are exogenously unprofitable to serve and the market may not be fully covered.
We impose the following restrictions on the cost differential \(c_h - c_l\) to ensure that an interior pure strategy equilibrium exists in any subgame in period 2, and on the proportion of high-cost customers \(\theta_h\) to ensure that an interior pure strategy equilibrium exists in period 1:

\[
c_h - c_l \leq 2 \quad \text{and} \quad \theta_h < \bar{\theta}_h \quad \text{if} \quad c_h - c_l > \frac{4}{3},
\]

where

\[
\bar{\theta}_h = \frac{-8 + 3(c_h - c_l) + \sqrt{9(c_h - c_l)^2 - 48(c_h - c_l) + 448}}{12(c_h - c_l)}.
\] (3)

### 3. Strategic Impact of Customer Base Composition

We examine the subgame equilibrium in period 2. Consider firm \(i\)'s customer base. Let \(\theta_{ij} = S_{ij}/\left(S_{ij} + S_{ij'}\right)\) be the proportion of type \(j\) customers at the start of period 2, where \(S_{ij}\) is defined previously in (2). Let \(\tilde{c}_i = \theta_{ij}c_h + \theta_{ij'}c_l\) be the average cost to serve. Customers choose firm \(i\) if \(\xi_{ij}^i - \xi_{ij'}^i \geq p_{ij2} - p_{ijR2}\). Since \(\xi_{ij2}^i - \xi_{ij'}^i\) is distributed uniformly over \([-1, 1]\), the proportion of type \(j\) customers retained by firm \(i\) is given by

\[
x_{ij2} = \Pr(\xi_{ij}^i - \xi_{ij'}^i \geq p_{ij2} - p_{ijR2}) = \frac{1}{2} + \frac{1}{2}(p_{ijR2} - p_{ij2}).
\] (4)

We solve for the equilibrium in Lemma B.1 in Appendix B and obtain

\[
p_{ij2} = 1 + \frac{1}{2}(\tilde{c}_i + c_l), \quad p_{ijR2} = 1 + \tilde{c}_i.
\] (5)

Firm \(i\) earns a profit of \(\pi_{ij}^O = p_{ij2} - c_l\) from a retained customer of type \(j\). In equilibrium, firm \(i\)'s ex ante expected profits from an existing customer in its customer base (accounting for the probability of retaining the customer) is given by

\[
\pi_{ij}^O x_{ij2} = \frac{1}{2}(1 + \frac{1}{2}(\tilde{c}_i - c_l))^2.
\] (6)

We find therefore that a firm’s expected profits are higher from a low-cost customer than from a high-cost customer (since \(c_h > \tilde{c}_i > c_l\)). Now, this might lead one to conclude that a firm would benefit more from an additional low-cost customer rather than an additional high-cost customer. Such a conclusion, however, turns out to be incorrect, for it overlooks the strategic impact of customer base composition.

#### 3.1. The Strategic Value of High-Cost Customers Under Information Asymmetry

Since the rival cannot discriminate among firm \(i\)'s customers, its price is based on the average cost to serve \(\bar{c}_i\), as shown in (4). Firm \(i\), on the other hand, tailors its price based on the individual customer’s cost to serve and offers a lower price to low-cost customers than to high-cost customers. Hence, the rival is more likely (than average) to acquire high-cost customers, who are less profitable, and is less likely to attract low-cost customers, who are more profitable. The reverse is true for firm \(i\) owing to its superior customer information. Thus, in equilibrium, firm \(i\) retains a better mix of customers and earns higher profits than the rival; as shown below:

\[
\Pi_{ij}^O = (S_{ij2} + S_{ijR2})[\frac{1}{2} + \frac{1}{8}\theta_{ij}\theta_{ij'}(c_h - c_l)^2],
\]

\[
\Pi_{ijR2}^O = (S_{ij2} + S_{ijR2})[\frac{1}{2} - \frac{1}{8}\theta_{ij}\theta_{ij'}(c_h - c_l)^2].
\] (7)

Furthermore, (7) shows that for a given customer base size, profits depend on the relative mix of customers. This gives rise to the possibility that a firm can leverage its superior customer information to gain a relative cost advantage by having an additional high-cost customer: Even though such a customer is equally costly for both firms, the rival is more likely to acquire and serve this customer. Moreover, an additional high-cost customer reduces the rival’s poaching intensity, which impacts the firm’s profits not just from the additional customer, but from all of its existing customers.

More formally, using the envelope theorem, the incremental impact of an additional customer of type \(j\) on firm \(i\)'s profits can be expressed as

\[
\frac{d\Pi_{ij}^O}{ds_{ij2}} = \frac{\partial \Pi_{ij}^O}{\partial s_{ij2}} + \frac{\partial \Pi_{ij}^O}{\partial p_{ij2}} \frac{\partial p_{ij2}}{ds_{ij2}} = \pi_{ij}^O x_{ij2} + \frac{1}{2} \frac{\partial p_{ij2}}{ds_{ij2}} [s_{ij2} \pi_{ij}^O + s_{ijR2} \pi_{ijR2}].
\] (8)

We note that the firm’s expected profits directly from the additional customer, given by \(\pi_{ij}^O x_{ij2}\), captures only a portion of this customer’s profit impact. The remaining impact is due to the strategic effect of this additional customer on the firm’s profitability from its other customers, which is given by the second term on the right-hand side of (8). The strategic effect itself consists of two components. One component, \(\partial p_{ij2}/\partial s_{ij2}\), is the impact of the additional customer on the rival’s equilibrium price. This is positive when the additional customer is a high-cost customer and negative in the case of a low-cost customer. The other component is the impact of an increase in the rival’s price on the firm’s profitability of its existing customers. This is always positive since the firm earns higher profits when the rival’s price is higher. Interestingly, when we consider the overall effect, we find that high-cost customers can be more valuable than low-cost customers under certain conditions.

\[\bar{\theta}_h\text{ is decreasing in } c_h - c_l; \bar{\theta}_h \approx 0.737 \text{ when } c_h - c_l = 2.\]
Lemma 1. For firm $i \in \{A, B\}$, $d\Pi^i_{12}/dS_{ih2} > d\Pi^i_{12}/dS_{il2}$ iff $\theta_{ih} < \frac{1}{2}$.

Proof. Refer to Appendix A.

This lemma shows that the difference in strategic effects dominates the difference in expected profits when the proportion of high-cost customers is sufficiently low. When $\theta_{ih} < \frac{1}{2}$, poaching intensity is relatively high. In this case, the relative impact of an additional customer on the intensity of poaching is more important than the relative profitability of the additional customer. In particular, when $\theta_{ih} < \frac{1}{2}$, an additional high-cost customer increases the uncertainty faced by the rival by making the customer base more heterogeneous and has a larger impact on the rival’s response. Consequently, a high-cost customer is more valuable than a low-cost customer since the strategic effect is more pronounced. In contrast, when $\theta_{ih} > \frac{1}{2}$, an additional high-cost customer reduces the uncertainty faced by the rival by making the customer base more homogeneous, and hence the strategic effect is less pronounced.

It is important to note that high-cost customers are more valuable not just because they reduce the rival’s poaching intensity, but also because the firm has superior information about its own customers such that it can retain a more favorable mix of low-cost customers. This information asymmetry effect is what makes it possible for the strategic effect to offset the lower profitability due to a high-cost customer. This can be shown by using the same model sans information asymmetry. As we show in Lemma B.2 in Appendix B, when both firms can perfectly discriminate among a firm’s customers or when neither firm has customer information to discriminate, an additional customer affects both firms equally, and customer base composition has no impact on firm profits.

Thus, based on the impact on a firm’s profits in its own customer base, having an additional high-cost customer can be more valuable. Similarly, considering the impact of an additional customer on the firm’s profits from the rival’s customer base, we show in Lemma B.3 in Appendix B that the firm’s profit is reduced to a lesser extent when one high-cost customer is removed from that customer base than when one low-cost customer is removed, if the proportion of low-cost customers in the rival’s customer base is sufficiently low ($\theta_{il} < \frac{1}{2}$). Consequently, we would expect that the total impact across both customer bases may favor having an additional high-cost customer if the proportion of high-cost customers in both customer bases is sufficiently low. We find that this is indeed the case. Since we will focus on the symmetric equilibrium of the overall game, we state the following result.

Proposition 1. When firms have symmetric customer bases ($S_{ij} = S_{ji} = \frac{1}{2} \theta$), it is more profitable for a firm to have an additional high-cost customer in its customer base than to have an additional low-cost customer, if and only if the proportion of high-cost customers is sufficiently low (iff $\theta_{h} < \frac{1}{2}$).

Proof. Refer to Appendix A.

Proposition 1 is in direct contrast to the conventional wisdom that customers who are less profitable are less valuable. Our finding suggests that in competitive settings a firm can benefit from strategically keeping its high-cost customers. It is this strategic insight that conventional wisdom overlooks when studying firms in isolation.

4. Managing Customers as Assets with Myopic Customers

Our analysis thus far has shown that the customer base composition at the start of period 2 has a strategic impact that may be counterintuitive. But this customer base composition is the outcome of the competitive interactions in period 1. Therefore, to understand how a firm may strategically manage the evolution of its customer base, we now go back to period 1 to solve for the symmetric subgame perfect equilibrium of the overall game. In period 1, the proportion of type $j$ customers retained by firm $i$ is given by

$$x_{ij1} = \Pr(s_{ij1} \in \{s_{ij1} \geq p_{ij1} - p_{R1}\}) = \frac{1}{2} + \frac{1}{2}(p_{ij1} - p_{R1}).$$

Lemma B.4 in Appendix B describes the period 1 equilibrium strategies from which we develop our key insights regarding the true value of a customer and the implications for customer retention.

4.1. Customer Profitability vs. Customer Value

Consider a customer of type $j$ who is in firm $i$’s customer base at the start of period 1 and is retained by firm $i$ in this period. Firm $i$’s expected profits from this customer are given by $\pi_{ij1}^{O} + \pi_{ij2}^{O} x_{ij2}$, which is the sum of the current period profits and the future expected profits and is the expected lifetime cash flow from this customer. This is conventionally regarded as the lifetime customer profitability when a firm is studied in isolation. However, this metric misses two aspects of a customer’s impact on a firm’s profitability. First, a customer who is not retained in period 1 may be acquired from the rival in period 2. The expected profits in this case is given by $\pi_{ij2}^{O}(1 - x_{ij2})$. Therefore, if firm $i$ were to retain an additional customer of type $j$ in period 1, the incremental profits from the customer per se, which we will refer to as $\text{incremental customer profitability}$ and denote by $l_{ij}$, are given by

$$l_{ij} = \pi_{ij1}^{O} + \pi_{ij2}^{O} x_{ij2} - \pi_{ij2}^{O}(1 - x_{ij2}).$$
The second aspect we need to consider is the strategic impact of an additional customer. As discussed in the previous section, this effect arises from the rival firm responding to the change in the firm’s customer base, which in turn impacts the firm’s overall profitability. Specifically, let $v_{ij}$ denote firm $i$’s customer value from retaining an additional customer of type $j$ in period 1, given by

$$v_{ij} = \pi^O_{ij} + \frac{d\Pi_i}{dS_{i\bar{j}}}.$$  

(11)

In equilibrium, substituting for $d\Pi_i/dS_{i\bar{j}}$ and using the envelope theorem, we obtain the following relationship between customer value $v_{ij}$ and incremental customer profitability $l_{ij}$:

$$v_{ij} = \pi^O_{ij} + \frac{d\Pi_i}{dS_{i\bar{j}}} = \pi^O_{ij} + \frac{\partial \Pi_i}{\partial S_{i\bar{j}}} + \frac{\partial \Pi_i}{\partial p_{i\bar{j}}} \frac{dp_{i\bar{j}}}{dS_{i\bar{j}}}$$

$$+ \sum_{j \in \{b, h\}} \frac{\partial \Pi_i}{\partial p_{ij}} \frac{dp_{ij}}{dS_{i\bar{j}}}$$

$$= \pi^O_{ij} + \pi^O_{i\bar{j}}x_{i\bar{j}} - \pi^O_{i\bar{j}}(1 - x_{i\bar{j}}) + \frac{\partial \Pi_i}{\partial p_{i\bar{j}}} \frac{dp_{i\bar{j}}}{dS_{i\bar{j}}}$$

$$+ \sum_{j \in \{b, h\}} \frac{\partial \Pi_i}{\partial p_{ij}} \frac{dp_{ij}}{dS_{i\bar{j}}}$$

$$= l_{ij} + \frac{1}{4}(c_j - \bar{c}),$$  

(12)

where, in the last step, we have substituted from (10) and for the period 2 equilibrium strategies from Lemma B.1 in Appendix B (when $\theta_i = \bar{\theta}_i$). Thus, customer value differs from incremental customer profitability because of the strategic impact of the additional customer on a firm’s profits from all other customers. In fact, in equilibrium, (12) shows that this strategic effect is always positive for a high-cost customer and negative for a low-cost customer.

**Proposition 2.** When customers are myopic, the incremental profits from retaining an additional high-cost customer (low-cost customer) always underestimate (overestimates) the value of retaining this customer.

**Proof.** The proof follows from (12).

Proposition 2 arises because, in equilibrium, an additional high-cost customer discourages poaching by the rival in the firm’s customer base more than it intensifies the rival’s retention efforts in the rival’s customer base, such that the overall strategic effect is positive. In contrast, an additional low-cost customer encourages poaching in the firm’s customer base more than it softens the rival’s retention efforts. This provides an interesting perspective to the current CRM practice. If a firm makes its retention decision based only on the incremental profitability of retaining a customer, it overlooks the impact of this action on the cash flows from the firm’s other customers due to the rival’s reaction. In other words, retaining an additional customer creates a competitive externality on the cash flows from other customers. Proposition 2 suggests that traditional metrics that do not capture this competitive externality systematically overestimate the true value of more profitable customers and underestimate the true value of less profitable ones. Therefore, traditional metrics may not be ideally suited for making retention decisions in a competitive context. Indeed, as we discuss next, when the right metric is used, even an unprofitable customer can add value to a firm.

**4.2. Implications for Customers Retention**

In making its retention decision, the firm must balance the value of retaining an additional customer with the costs of doing so, namely, lowering its price to retain the additional customer. At the optimal retention price, we have, from the first-order conditions,

$$\frac{d(\Pi_i + \Pi_{i\bar{j}})}{dp_{ij}} = 0$$

$$\Rightarrow \frac{d(S_{ij} \pi^O_{ij} x_{ij})}{dp_{ij}} + \frac{d\Pi_i}{dS_{i\bar{j}}} \frac{dp_{i\bar{j}}}{dp_{ij}} = 0,$$

$$\Rightarrow S_{ij} x_{ij} - \frac{1}{2} S_{ij} \left( \pi^O_{ij} + \frac{d\Pi_i}{dS_{i\bar{j}}} \right) \frac{dp_{i\bar{j}}}{dp_{ij}} = 0,$$

$$\Rightarrow x_{ij} = \frac{1}{2} v_{ij},$$  

(13)

where we have substituted $dS_{i\bar{j}}/dp_{ij} = S_{ij}(dx_{ij}/dp_{ij})$ from (2) and $dx_{ij}/dp_{ij} = \frac{1}{2}$ from (9). Therefore, our analysis shows that customer value, and not customer profitability, is the appropriate metric to use for retention decisions: In (13), a firm’s retention rate $x_{ij}$ is in direct proportion to customer value $v_{ij}$ and is not driven by customer profitability per se. In addition, customer value behaves differently than customer profitability, leading to some interesting and important implications for the firm’s retention strategy.

To begin with, we find that some customers may become more valuable to retain when their cost to serve is higher even though this makes them less profitable. As a result, contrary to conventional advice, the firm may offer them a lower price and retain them to a greater extent than when their cost to serve is lower. This is illustrated for the case of high-cost customers in the numerical examples in Table 3, where customer value and retention for high-cost customers is increasing in $c_i$ even though their profitability is decreasing. Furthermore, the price offered to...
them is decreasing in $c_h$; that is, the firm is also more aggressive in retaining high-cost customers when they are less profitable.

Intuitively, when $c_h$ is higher, high-cost customers are less profitable. But they are also more effective in discouraging poaching, and thus their competitive externality is higher. The balance between these two effects determines the net impact on customer value and retention. In Proposition 1, we had shown that the competitive externality effect can offset the effect of lower customer profitability in period 2 when $\theta_h$ is sufficiently low. We now find that the competitive externality effect can further offset the effect of lower customer profitability in period 1. This occurs when $\theta_l$ is even lower, such that the competitive externality is more pronounced, and if high-cost customers are sufficiently more costly to serve, such that their expected profits are lower in magnitude and have a smaller impact on customer value. As a result, in such instances, customer value is higher when cost to serve is higher, which also motivates the firm to compete more intensely for such customers.

Furthermore, a firm may retain high-cost customers even when they are unprofitable in terms of their expected lifetime cash flow. This is also illustrated in the numerical examples in Table 3. In general, high-cost customers are less profitable when $c_h$ is high (since they are costlier to serve) and $\theta_h$ is low (since competition is more intense). We find that they are in fact unprofitable when $c_h$ is sufficiently high and $\theta_l$ is sufficiently low. But the competitive externality is also more pronounced in such instances and offsets the effect of customer profitability. Hence, a firm actively competes to retain such unprofitable customers since their value is positive. The following proposition summarizes our findings.

**Proposition 3.** When customers are myopic,
(i) a firm retains unprofitable high-cost customers iff the cost differential between high-cost and low-cost customers is sufficiently high and the proportion of high-cost customers is sufficiently low (iff $c_h - c_l > \frac{2}{3}$ and $\theta_h < \frac{1}{2} - 2/(3(c_h - c_l))$).
(ii) a firm retains more high-cost customers when their cost to serve is higher and offers them a lower price, iff the cost differential between high-cost and low-cost customers is sufficiently high and the proportion of high-cost customers is sufficiently low (iff $c_h - c_l > \frac{2}{3}$ and $\theta_h < \frac{1}{2} - 2/(3(c_h - c_l)))$.

**Proof.** Refer to Appendix A.

This proposition suggests a different CRM strategy from the conventional advice of turning unprofitable customers into profitable ones or making them leave. We find that a profit-maximizing firm may retain unprofitable customers and keep them unprofitable by not raising their price. In other words, it can be profitable to “poison” one’s own customer base. Our findings might explain, for instance, why some banks became reluctant to alienate their unprofitable customers and instead actively competed for them (as reflected in the opening quote in §1). Our results further suggest that a segmentation mindset to managing customers, wherein firms instinctively keep “good” customers and prune the “bad” ones, may not always be appropriate in competitive settings.

Instead, firms should adopt a customer portfolio mindset that recognizes that the value of customers is interlinked because of the competitive externality. Different customers may play different roles in determining the overall profitability of the customer base. A firm should therefore balance individual customer profitability with the strategic impact of its customer base composition.

### 5. Implications When Customers Are Forward Looking

We now examine whether our main insights continue to apply when customers are forward looking. At the outset, we acknowledge that our analysis is limited by the necessity to conduct numerical analysis for some of our results. Therefore, our results will have to be interpreted with appropriate caution.

When customers are forward looking, they form rational expectations regarding period 2 prices based on the period 1 prices they observe. Since we are interested in a market context where a firm typically deals with its existing customers on a one-to-one basis and customizes the incentives it offers, we assume

<table>
<thead>
<tr>
<th>$c_h$</th>
<th>Price to existing customers</th>
<th>Proportion of customer retained</th>
<th>Expected lifetime profitability</th>
<th>Customer value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{H}$</td>
<td>$\text{L}$</td>
<td>$\text{H}$</td>
<td>$\text{L}$</td>
</tr>
<tr>
<td>1.6</td>
<td>1.448</td>
<td>1.032</td>
<td>0.3128</td>
<td>0.5208</td>
</tr>
<tr>
<td>1.8</td>
<td>1.443</td>
<td>1.029</td>
<td>0.3137</td>
<td>0.5207</td>
</tr>
<tr>
<td>2</td>
<td>1.425</td>
<td>1.025</td>
<td>0.3200</td>
<td>0.5200</td>
</tr>
</tbody>
</table>

*Note. In this example, $\theta_h = \frac{c_h}{c_l}$, $c_l = 0$, and $\delta = 0$; H denotes high-cost customers and L denotes low-cost customers.*
that the personalized price for a particular segment is not publicly announced and is known only to customers in that segment. In practice, firms may classify customers into several tens of different segments based on their profit potential (e.g., Hartfeil 1996, Selden and Colvin 2003), making it impractical for a customer to monitor each firm’s specific incentives to each of the different segments. In contrast, since a firm charges the same price to all of its rival’s customers, we assume that it is publicly known to all customers in the market. We assume that customers hold rational beliefs about prices that they do not observe (that will be confirmed in equilibrium). The implication of our assumptions is that a customer’s belief about future prices can be affected only by changes in prices that she observes. We assume that $c$, $\theta$, and the distribution of $\xi$ are common knowledge to customers. Let $\delta \in [0,1]$ denote their discount factor. Because of space constraints, we only provide an overview of our analysis. The details can be found in §C of the online supplemental appendix.

Period 2 interactions are as in the main model. Therefore, the results of Proposition 1 hold as before. In particular, the strategic effects that we identified arise only in period 2 and are not affected. Let $W_{ij}$ denote a type $j$ customer’s ex ante expected utility (prior to preference shocks) of being in firm $i$’s customer base at the start of period 2. From (C.3) in the online supplemental appendix, we have

$$W_{ij} = V_i - p_{ij} + (1 - x_{ij})^2,$$

where $x_{ij}$ is given by (4). In period 1, in firm $i$’s customer base, a type $j$ customer chooses firm $i$ iff

$$V_i - p_{ij} + \xi_{ij} + \delta W_{ij} > V_j - p_{ij} + \xi_{ij} + \delta W_{ij},$$

Therefore, firm $i$’s retention rate is given by

$$x_{ij} = \Pr\{V_i - p_{ij} + \xi_{ij} + \delta W_{ij} > V_j - p_{ij} + \xi_{ij} + \delta W_{ij}\},$$

We note that the period 2 equilibrium prices and retention rates in (15) are a function of period 1 retention rates. Thus, similar to Shin and Sudhir (2010), the first period retention rates are only implicitly defined, and it is not possible to obtain an explicit solution. We follow the approach in Shin and Sudhir (2010) to derive the symmetric equilibrium. We are, however, only able to numerically verify that the results of Proposition 3 hold. Since the strategic effects we previously uncovered arise only in period 2, the results of Proposition 2 are not affected by forward-looking behavior and hold as before. The main difference when customers are forward looking is that the results of Proposition 3 hold over a narrower range of parameters. This is because forward-looking behavior affects customer price sensitivities as explained below.

In equilibrium, we find that high-cost customers are less price sensitive to their current firm’s price than in the myopic case. For instance, when firm $A$ raises its price to high-cost customers, they rationally anticipate that firm $A$’s customer base in period 2 would have fewer high-cost customers. This would cause period 2 prices to be lower in firm $A$’s customer base (since prices are increasing in the average cost to serve). Correspondingly, firm $B$’s customer base in period 2 would have more high-cost customers and higher prices. Consequently, it becomes less attractive to switch to firm $B$ in response to firm $A$’s price increase. Similarly, low-cost customers are more price sensitive to their current firm’s price than in the myopic case. Therefore, compared to the case with myopic customers, firms charge their high-cost customers a higher price because they are less price-sensitive, which reduces the range of parameters where retained high-cost customers are unprofitable.

6. Conclusion

Our analysis shows that in a competitive setting, when a firm has superior information about its customers compared to its rival, customer base composition affects the profitability of the overall customer base by influencing the rival’s poaching intensity. Under certain conditions, this may lead to qualitatively different prescriptions than conventional advice and have implications for measuring customer value. The key message from our analysis is that retaining high-cost-to-serve customers, even when they are unprofitable, can help increase the overall profitability of the firm. This is because in a competitive context, the true value of a high-cost customer cannot be captured by customer lifetime profitability alone. The right metric to use must account for the competitive externality that each customer imposes on the profitability of other customers. This further suggests that firms may need to shift from a traditional segmentation mindset, which views each customer in isolation, to a customer portfolio mindset, which recognizes that the value of customers is interlinked, and that different customers may play different roles in determining the overall profitability of the customer base.

In §D of the online supplemental appendix, we show that our analysis can be extended in several directions: when there are three customer cost-to-serve types, when high-cost customers are intrinsically

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13 This can be seen from (2), (4), and (5). Also, (C.5)–(C.7) in §C of the online supplemental appendix express period 2 variables as functions of period 1 retention rates.
unprofitable, when high-cost customers are more price sensitive than low-cost customers (rather than being equally price sensitive), when firms keep track of past customers, and when customers have persistent preferences (loyalty) for one of the firms. Our analysis shows that our main insights continue to hold qualitatively in these settings. We now briefly discuss some limitations of our research.

First, the strategic effects we identify do not arise in a monopoly setting, and our results apply only to settings where there is sufficient scope for competitive customer poaching. Second, our findings provide one possible rationale for why some firms may retain their unprofitable customers. But there could be others in practice, such as avoiding negative word of mouth or bad publicity that might be generated by alienating customers. Third, we do not examine firms’ incentives to acquire the capability to discriminate between customers. In particular, it may be interesting to examine whether and when only one of the firms in a market may acquire this capability (e.g., Pazgal and Soberman 2008). Moreover, to parsimoniously capture the notion that a firm has better information about its customers than the rival, we assumed that a firm can perfectly discriminate among its customers. However, it is also possible that a firm’s customer information is imperfect to begin with and the firm is able to learn more about its customers and target them better the longer it serves them. This may cause firms to retain customers including those that may initially appear to be high cost. We do not examine this aspect.

Finally, we examined the implications when forward-looking customers modify their purchase behavior taking into account how revealing their type to the rival firm will impact the evolution of the firm’s customer base and, therefore, the future prices. However, in some settings, customers may also modify their behavior to hide their type from their current firm, provided it is not too costly for them to do so. For instance, Shin et al. (2012) analyze a monopoly setting where forward-looking customers can endogenously decide their cost type by choosing the level of service to demand. They find that such customers may reduce their service demand or abstain from making a purchase if they believe that higher service demand may lead to higher future prices, which may prevent the firm from learning the customer’s true type. Thus, there can be asymmetric information not only between firms, but also between customers and their current firm. It is beyond the scope of our current work to simultaneously consider both dimensions of asymmetric information. As such, our results may be more relevant when there is a higher degree of asymmetric information between firms than between customers and their current firm, for instance, when most customers may find it costly to significantly disguise their types (i.e., the disutility of altering their innate behavior is high).

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Appendix A. Proofs of Propositions and Lemmas

Proof of Lemma 1. From (7) we have,
\[
\frac{d\Pi_0}{dS_{i2}} = \frac{1}{2} + \frac{1}{8}(c_i - c_j)^2,
\]
which is positive iff \(c_i > \frac{1}{4}(c_h + c_j)\) or \(\theta_{i1} > \frac{1}{2}\). \(\square\)

Proof of Proposition 1. When \(\theta_{i1} = \theta_{j1}\), we have, from (A1) and (B2),
\[
\frac{d\Pi_{i2}}{dS_{i2}} - \frac{d\Pi_{j2}}{dS_{j2}} = \frac{3}{8}(2c_i - c_h)(c_h - c_j),
\]
which is positive iff \(c_i > \frac{1}{2}(c_h + c_j)\) or \(\theta_{i1} > \frac{1}{2}\). \(\square\)

Proof of Proposition 3. The expected lifetime profits from a high-cost customer retained by firm \(i\) in period 1 is given by
\[
\frac{1}{16}(24 - (c_h - c_j)(1 - \theta_h)(16 + (c_h - c_j)(1 + 2\theta_h))),
\]
which is a quadratic convex expression in \(\theta_h\) and increasing \(\theta_h\). It follows that it is negative iff \(c_h - c_j > 2\sqrt{22} - 8\) and \(\theta_h < (c_h - c_j - 16 + \sqrt{64 + 9(2c_i - c_j) + 9(c_h - c_i)^2})/(4(c_h - c_j))\). For firm \(i\)’s retention rate of high-cost customers, we have
\[
x_{i|1} = \frac{1}{2} - \frac{1}{4}(c_h - c_j)(1 - \theta_h)
\]
and
\[
x_{i|1} = \frac{1}{4}(1 - \theta_h)
\]
which is positive if \(c_h - c_j > \frac{1}{2}\) and \(\theta_h < \frac{1}{2} - 4/(3(c_h - c_i))\). We also have
\[
\frac{dp_{i|1}}{dc_h} = \frac{1}{2}(1 + \theta_h) - \frac{1}{2}(c_h - c_j)(1 - \theta_h),
\]
which is negative whenever \(c_h - c_j > \frac{1}{2}\) and \(\theta_h < \frac{1}{2} - 4/(3(c_h - c_i))\). \(\square\)
Appendix B. Additional Lemmas

**Lemma B.1.** In period 2, in firm $i$’s customer base, equilibrium prices are given by

$$ p_{ij}^2 = 1 + \frac{1}{2}(c_i + c_j), \quad p_{ij}|_{\bar{R}_2} = 1 + \bar{c}_i. $$

**Proof.** Firm profits in firm $i$’s customer base are given by

$$ \Pi_i^2 = \sum_{j \in [i, \bar{R}]} S_{ij}x_{ij}(p_{ij} - c), $$

$$ \Pi_{ij}^2 = \sum_{j \in [i, \bar{R}]} S_{ij}(1 - x_{ij})(p_{ij} - c_j), \quad (B1) $$

where $x_{ij}$ is given by (4). Solving for the profit maximizing prices, we obtain the desired result. \( \square \)

**Lemma B.2.** In period 2, in firm $i$’s customer base, when neither firm has customer information or when both firms have customer information, customer base composition does not impact firm profits.

**Proof.** When neither firm has information about customer types, let $p_{ij}^k$ be firm $i$’s price to a customer of type $j$ in its rival’s customer base. Solving for the equilibrium prices in firm $i$’s customer base, we obtain $p_{ij}^k = p_{ij}^k = 1 + c_j$. Both firms earn a profit $\frac{1}{2}(S_{ij} + S_{ij})$, which is independent of the customer base composition (i.e., the relative proportion of high-cost and low-cost customers). When both firms are informed about customer types, let $p_{ij}^k$ be firm $i$’s price to a customer of type $j$ in its rival’s customer base. Solving for the equilibrium prices in firm $i$’s customer base, we obtain $p_{ij}^k = p_{ij}^k = 1 + c_j$. Both firms earn a profit $\frac{1}{2}(S_{ij} + S_{ij})$, which is independent of the customer base composition.

**Lemma B.3.** For firm $i \in \{A, B\}$, $d\Pi_i^2/dS_{ij} > d\Pi_j^2/dS_{ij}$ iff $\theta_j \in (0, \frac{1}{2})$.

**Proof.** From (7) we have

$$ d\Pi_i^2/dS_{ij} = \frac{1}{2} \bar{c}_j - \frac{1}{4}(\bar{c}_j - c_j)^2, $$

$$ d\Pi_j^2/dS_{ij} = \frac{1}{2} \bar{c}_j - \frac{1}{4}(\bar{c}_j - c_j)^2, $$

$$ = \frac{1}{2} (2\bar{c}_j - c_j)(c_j - c_j), $$

which is positive iff $\bar{c}_j > \frac{1}{2}(c_j + c_j)$ or $\theta_j > \frac{1}{2}$. We note that in this case, using the envelope theorem, we can express the incremental impact due to an additional customer of type $j$ in firm $i$’s customer base on firm $i$’s profits from its rival’s customer base as follows:

$$ \frac{d\Pi_i^2}{dS_{ij}} = \frac{d\Pi_j^2}{dS_{ij}} + \sum_{j \in [i, \bar{R}]} \frac{d\Pi_j^2}{dS_{ij}} \frac{dp_{ij}^2}{dS_{ij}}/dS_{ij}, $$

$$ = \pi_{ij}(1 - x_{ij}) + \frac{1}{2} \sum_{j \in [i, \bar{R}]} S_{ij} \pi_{ij} \frac{dp_{ij}^2}{dS_{ij}}/dS_{ij}. \quad (B2) $$

The first term on the right-hand side of (B3) corresponds to the direct expected profits firm $i$ stands to lose from the one less customer per se. This favors having one less high-cost customer since they are less profitable. The second term captures the strategic effect of this one less customer on firm $i$’s overall profits from the rival’s customer base. The strategic effect of one less high-cost customer in the rival’s customer base is negative: when there is one less high-cost customer, firm $i$ poaches more intensely, which induces the rival to intensify its retention efforts by reducing its prices to both customer types, thereby negatively impacting firm $i$’s profits from all its acquired customers. In contrast, the strategic effect of one less low-cost customer is positive since this causes the rival to soften its retention efforts. \( \square \)

**Lemma B.4.** When customers are myopic, there is a unique symmetric equilibrium of the overall game. Firm $i$’s period 1 equilibrium prices are given by

$$ p_{ii}^1 = 1 + c_i - \frac{1}{2}\theta_i \Delta_i(8 + 3\Delta_i), $$

$$ p_{ii}^1 = 1 + c_i + \frac{1}{2}\theta_i \Delta_i(8 - 3\Delta_i), $$

$$ p_{ir}^1 = 1 + c_i + \frac{1}{2}\theta_i \Delta_i(8 - 3\Delta_i) - \frac{1}{8}\theta_i \Delta_i^2, $$

$$ p_{ir}^1 = 1 + c_i + \frac{1}{2}\theta_i \Delta_i(8 - 3\Delta_i) + \frac{1}{8}\theta_i \Delta_i^2, $$

where $\Delta_i = c_b - c_i$.

**Proof.** Period 1 firm profits in firm $i$’s customer base are given by

$$ \Pi_i^1 = \sum_{j \in [i, \bar{R}]} \frac{1}{2}\theta_j x_{ij}(p_{ij} - c), $$

$$ \Pi_{ij}^1 = \sum_{j \in [i, \bar{R}]} \frac{1}{2}\theta_j (1 - x_{ij})(p_{ij} - c_j). \quad (B4) $$

From Lemma B.1, we obtain firm $i$’s period 2 profits as

$$ \Pi_{i2} = (S_{ij} + S_{ij})[\frac{1}{2} + \frac{1}{2}\theta_i \theta_j \Delta_i^2] + (S_{ij} + S_{ij})[\frac{1}{2} + \frac{1}{2}\theta_i \theta_j \Delta_i^2]. \quad (B5) $$

We first show that there cannot be a symmetric equilibrium where a firm captures all of the rival’s high-cost customers. Suppose, toward a contradiction, such an equilibrium existed. Solving the first-order conditions for such an equilibrium, we obtain

$$ p_{ii1} = 3 + \theta_b - \frac{3}{8}\Delta_b^2 \theta_i^2, $$

$$ p_{ii1} = 3 + \theta_b - \frac{3}{8}\Delta_b^2 \theta_i^2, $$

$$ p_{ii1} = 4 + 2\theta_i - \frac{3}{8}\Delta_i^2 \theta_i^2, $$

$$ p_{ii1} = 4 + 2\theta_i - \frac{3}{8}\Delta_i^2 \theta_i^2. \quad (B6) $$

But given these prices, it is straightforward to verify that say firm 1 has an incentive to set $p_{ij}^1$ such that $x_{ij} > 0$. Similarly, there cannot be a symmetric equilibrium where a firm captures all of the rival’s low-cost customers. Thus, we are left with the possibility of an interior equilibrium. The first-order conditions in this case can be written as

$$ \frac{d\Pi_i^1 + \Pi_{ij}}{dp_{ij1}} = 0, $$

$$ \Rightarrow \frac{d\Pi_i^1}{dp_{ij1}} + \frac{d\Pi_{ij}}{dp_{ij1}} = 0, $$

$$ \Rightarrow \frac{d\Pi_{ij}}{dp_{ij1}} = 0, $$

$$ \Rightarrow \frac{\pi_{ij}(1 - x_{ij})}{2} (p_{ij} - c_j) - \frac{1}{2} \frac{d\Pi_{ij}}{dp_{ij1}} = 0, \quad (B7) $$

$$ \Rightarrow \sum_{j \in [i, \bar{R}]} \theta_j [(1 - x_{ij}) - \frac{1}{2}(p_{ij} - c_j) - \frac{1}{2} \frac{d\Pi_{ij}}{dp_{ij1}}] = 0, \quad (B8) $$

When evaluating $d\Pi_i^2/dS_{ij}$, we have substituted $S_{ij} = \theta_j - S_{ij}$. 

\[ \frac{d\Pi_i^1}{dp_{ij1}} = 0. \]
where we have substituted $dS_{ij}/dp_{A1} = dS_{ij}/dp_{R1} = -\frac{1}{2} \theta_i$ from (2) and (9). In symmetric equilibrium, $p_{A1} = p_{R1}$ and $p_{AR1} = p_{BR1}$, and the above conditions reduce to

\begin{equation}
1 + p_{AR1} - 2p_{A1} + c_i - \frac{1}{2} (1 - \theta_i) \Delta_i^2 = 0, \tag{B9}
\end{equation}

\begin{equation}
1 + \theta_i p_{A1} + \theta_i p_{AR1} - 2p_{AR1} + c_i - \frac{1}{2} \theta_i \Delta_i^2 = 0. \tag{B10}
\end{equation}

Solving the above linear equations, we obtain the desired equilibrium strategies.

Although it is straightforward to verify that the second-order conditions are met, $\Pi_i$ may not be concave for some off-equilibrium prices. We show that we can construct a quadratic concave upper bound $\Pi_i$ for $\Pi_j$, with the same optimum point and optimum value as $\Pi_i$. Therefore, there is no profitable deviation for firm $i$ from the candidate equilibrium strategy. Given, say, firm $B$’s equilibrium prices for the price range when $x_{ij} \in [0, 1]$, we can express $\Pi_i$ as

$$\Pi_i(d_{ij}, d_{j1}, d_{k}) = \Pi_i - \frac{1}{4} (\theta_i d_{ij}^2 + \theta_i d_{j1}^2 + d_{k}^2)$$

$$+ \frac{1}{32} \theta_i^2 d_{ij}^2 \Delta_i (d_{ij} - d_{j1})^2 \left[ \frac{2}{2 - (\theta_i d_{ij} + \theta_i d_{j1} + d_{k})} - \frac{1}{2 + (\theta_i d_{ij} + \theta_i d_{j1} + d_{k})} \right], \tag{B11}$$

where $d_i = p_{A1} - p_{AR1}$, $d_k = p_{A1} - p_{BR1}$ denote the deviation from firm $A$’s candidate equilibrium prices $(p_{A1}', p_{AR1}', p_{BR1}')$, and $\Pi_i$ is firm $A$’s profits at the candidate equilibrium point. To construct an upper bound, we note that the second term within the square brackets in (B11) can be dropped: the denominator of this term is strictly positive since it is the size of firm $A$’s period 2 customer base multiplied by 4. The first term within the square brackets is strictly increasing in the price deviations and attains its maximum when the price deviations are at their largest values, i.e., when $x_{A1} = 1$ and $x_{B1} = 0$. We bound this term by substituting its maximum value. Let $\bar{\Pi}_i$ denote the upper bound for $\Pi_i$ constructed in this manner, by

$$\bar{\Pi}_i(d_{ij}, d_{j1}, d_{k}) = \Pi_i - \frac{1}{4} (\theta_i d_{ij}^2 + \theta_i d_{j1}^2 + d_{k}^2)$$

$$+ \frac{1}{8 - 3\Delta_i (1 - 2\theta_i)} \theta_i^2 d_{ij}^2 \Delta_i (d_{ij} - d_{j1})^2. \tag{B12}$$

We note that $\bar{\Pi}_i$ is a strictly concave quadratic function of the price deviations that attains its maximum of $\Pi_i$ when the price deviations are zero. Therefore, $\bar{\Pi}_i$ too attains its maximum when the price deviations are zero. Concave upper bounds can be constructed in a similar manner for price deviations corresponding to when $x_{A1}$, $x_{B1} \in [0, 1]$, $x_{A1} = 1$, and when $x_{A1}$, $x_{B1} \in [0, 1]$, $x_{B1} = 0$.

To verify that the boundary conditions $x_{ij} \in (0, 1)$ are satisfied when (2.2) holds, we note that

$$x_{ij} = \frac{1}{2} + \frac{1}{2} \Delta_i \theta_i - \frac{1}{32} \Delta^2 \theta_i (1 - 2 \theta_i), \tag{B13}$$

which is quadratic in $\Delta_i$, and strictly positive for $\Delta_i = 0$ and $\Delta_i = 2$. Furthermore, from Descartes’ sign-change rule, the above expression has at most one positive root. Thus, $x_{ij}$ is strictly positive over the entire parameter range. We also note that the above expression is a convex, strictly increasing function of $\theta_i$. Substituting $\theta_i = 1$, $x_{ij}$ is bounded above by $(1 + 8 \Delta_i + 3 \Delta_i^2)/16$, which is not greater than 1 for $\Delta_i \leq 4$. For $\Delta_i \in (4, 2)$, substituting $\theta_i = (-8 + 3 \Delta_i + \sqrt{9 \Delta_i^2 - 48 \Delta_i + 48})/(12 \Delta_i)$, $x_{ij}$ is bounded above by 1. Similarly, we verify that $x_{ij} \in (0, 1)$ in equilibrium. □

References


