Learning and Teaching as Communicative Actions: Improving Historical Knowledge and Cognition Through Second Life Avatar Role Play

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When Does It Pay to Delay Supplier Qualification?
Theory and Experiments

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We study a procurement setting in which the buyer seeks a low price but will not allocate the contract to a supplier who has not passed qualification screening. Qualification screening is costly for the buyer, involving product tests, site visits, and interviews. In addition to a qualified incumbent supplier, the buyer has an entrant of unknown qualification. The buyer wishes to run a price-only, open-descending reverse auction between the incumbent and the entrant, and faces a strategic choice about whether to perform qualification screening on the entrant before or after the auction. We analytically study the buyer’s optimal strategy, accounting for the fact that under postauction qualification, the incumbent knows he could lose the auction but still win the contract. In our analysis, we derive the incumbent’s optimal bidding strategy under postauction qualification and find that he follows a threshold structure in which high-cost incumbents hold back on bidding—or even boycott the auction—to preserve their profit margin, and only lower-cost incumbents bid to win. These results are strikingly different from the usual open-descending auction analysis where all bidders are fully qualified and bidding to win is always a dominant strategy. We test our analytical results in the laboratory, with human subjects. We find that qualitatively our theoretical predictions hold up quite well, although incumbent suppliers bid somewhat more aggressively than the theory predicts, making buyers more inclined to use postauction qualification.

Key words: procurement auctions; supplier qualification; experiments

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1. Introduction

Procurement is important—the average U.S. manufacturer spends roughly 60% of its revenue buying goods and services (U.S. Department of Commerce 2005). Competitive bidding (auctions) is a powerful tool used by many buyers to manage spending. Seeking price concessions from incumbent suppliers, it is common for buyers to have a highly motivated new (entrant) supplier compete in a reverse auction against the incumbent supplier when the incumbent’s contract is up for renewal.

Although contract price is a concern, buyers generally will not switch the contract to an entrant unless the entrant has been verified to be fully qualified for the business. Performing qualification screening on a supplier is the act of verifying that the supplier is indeed able to comply with all the contract specifications (e.g., on product, delivery, packaging) with a reasonable degree of certainty. The process of screening an entrant’s qualifications is costly for the buyer, and can include testing products, visiting production facilities, verifying surge capacity availability, etc.

When it comes to screening an entrant’s qualifications, the buyer has a strategic choice: attempt to qualify the entrant prior to the auction (“prequalification”), or delay qualification screening until after the auction (“postqualification”). Both approaches are used in practice; for example, Beall et al. (2003) discuss cases where Bechtel (a large engineering firm) followed a prequalification approach, whereas Kulp and Randall (2005) discuss how a pharmaceutical company procuring industrial chemicals allowed bids by entrants whose qualifications had not yet been verified. Both approaches have their respective advantages. Under postqualification, the buyer attempts to screen entrants only if their eventual bid is sufficiently attractive. This allows the buyer to directly hold an auction and saves money that might have otherwise been wasted performing qualification screening on an entrant whose bid turns out not to be competitive.
relative to the incumbent’s bid. On the other hand, if an auction occurs under prequalification, the buyer commits to awarding the contract to the lowest bidder, which can push the incumbent supplier to bid more aggressively.

Under postqualification, the entrant supplier might win the auction but fail qualification screening afterward, forcing the buyer to award the contract to the incumbent supplier (who lost the auction), paying a higher bid price. Standard auctions assume that the bid-taker commits to awarding the contract to the bidder who submitted the best bid, and most of the existing theoretical models of auctions also make this assumption, as do laboratory experiments that test these standard models. In contrast, procurement auctions in practice are oftentimes buyer determined (such as with postqualification), meaning that the buyer reserves the right to select the winner after the auction (see, for example, Jap 2002), so a supplier may lose the auction but win the contract. Buyer-determined, nonbinding auctions have not yet been extensively studied (two exceptions, Engelbrecht-Wiggans et al. 2007 and Wan and Beil 2009, are discussed in the literature review). The question of the optimal bidding strategy in an open-bid auction where a bidder may lose the auction (by not submitting the lowest bid) but yet win the contract, is difficult analytically, and we are the first to tackle it. We derive new theoretical results for this setting from both the buyer and bidder perspective, and experimentally test our theory’s predictions using a controlled laboratory experiment with human subjects.

To enable both theoretical and experimental analyses, we study a stylized problem that captures the salient features of the buyer’s pre- or postqualification decision. In our model, the buyer has an expiring contract with her incumbent supplier and, approached by a new entrant supplier, wishes to conduct an open-descending procurement auction between the incumbent and the entrant. The buyer can choose to screen the entrant using prequalification; if the entrant successfully passes it, the buyer can then hold a binding auction in which the low bid wins the contract. However, prequalification may backfire on the buyer if the entrant fails to be qualified—in such a case the buyer not only wastes the qualification cost but also loses the opportunity to run the auction, forcing her to renew the incumbent’s contract without any reduction in price. Alternatively, the buyer can choose to use postqualification screening on the entrant and screen the entrant only if the entrant wins the auction. In this case, the incumbent knows that he could lose the auction but still win the contract if the entrant fails postqualification. We address the following questions:

Research Question 1. What is the incumbent’s optimal bidding strategy under postqualification?

Research Question 2. How does the answer to Question 1 depend on the probability that the entrant is truly qualified, the buyer’s cost of performing qualification screening on the entrant, and the auction reserve price?

Research Question 3. Under what circumstances will the buyer prefer to use postqualification?

Research Question 4. To what extent does the theory we develop to answer the above questions predict subjects’ behavior in controlled laboratory experiments?

Answers to our research questions are summarized in §7. Section 2 reviews related literature, followed by the model description in §3. Section 4 provides theoretical analyses, and §§5 and 6 describe our experimental design and results, respectively. Section 7 concludes.

2. Literature Review

Elmaghraby (2000) provides a detailed review of procurement work in the operations and economics literature. Many such papers, including ours, apply auctions as the means of price discovery during the procurement process. Krishna (2009) provides excellent treatments and references on auctions.

Our paper studies how supplier qualification screening manifests itself in the auction-bidding behavior of entrants and incumbents. Although supplier qualification is common in practice, surprisingly little has been written about it in the procurement auction literature. To our knowledge, only one other paper studies supplier qualification in the context of procurement auctions. That paper, Wan and Beil (2009), focuses on the buyer’s optimal mechanism design problem for a setting with an unconstrained number of bidders whose qualification probabilities and costs are ex ante symmetric. Optimal mechanism design searches among all possible mechanisms and finds the one that maximizes the objective function of the designer. Bidder incentives are captured as constraints on an optimization problem, whose solution provides a theoretical benchmark. In the present paper, our focus is different. We study a setting with two suppliers, an incumbent and entrant, who are ex ante asymmetric in cost and qualification probability. For this setting, we focus on theoretical and experimental results. We do so for the reverse open-descending, price-only auction, where our reasons for examining this mechanism are twofold: First, reverse open-descending, price-only auctions are commonly used for procurement in practice, making it important to study such mechanisms. Second, the simplicity and practicality that make such mechanisms widely used in practice also make them amenable to study in the lab, a major purpose of the present paper.

In addition to providing the first equilibrium analyses for bidding behavior in reverse open-descending,
price-only auctions with possibly unqualified bidders, in testing these predictions in the lab, the present paper is also the first experimental study of auctions with possibly unqualified bidders. Laboratory experiments testing various aspects of auction theory go back to the early 1980s. Most early work focused on testing revenue equivalence among various auction formats and exploring possible explanations for its failure, as well as on investigating the winner’s curse in common value auctions; see Kagel (1995) for a review of experimental auctions work prior to 1995 and Kagel and Levin (2002) for a review of lab experiments in the common and affiliated value settings. Experimental work on procurement auctions focused on settings where price is not the only attribute of interest (our paper falls into this category). Engelbrecht-Wiggans and Katok (2006) compare auctions with noncompetitive contracts. Bichler (2000) was the first to test multiattribute auctions in the laboratory. Chen-Ritzo et al. (2005) compare a multiattribute procurement auction to a price-only auction and demonstrate that a multiattribute auction can be more efficient. Engelbrecht-Wiggans et al. (2007) study a setting where suppliers have nonprice attributes, but the auction is conducted on price, and explain when the buyer is better off committing to award the contract on price alone, ignoring the nonprice attributes. We analyze a different (and essential) nonprice attribute, supplier qualification, where the buyer seeks to award the contract to the lowest-priced qualified supplier.

In the present paper, the incumbent, who is known to the buyer, has already passed qualification screening. The entrant, who is unknown to the buyer, has not yet been qualified. There is other theoretical work that model different features that make incumbent and entrant suppliers unalike. Zhou (2003) models informational differences between incumbents and entrants when the cost of the contract is highly uncertain, finding that the incumbent generally bids aggressively owing to his informational advantage. In contrast, we find that the incumbent’s qualification “advantage” in our setting can cause him to bid much less aggressively. More generally, incumbent or entrant status can motivate studying auctions where bidders possess asymmetric cost distributions; for a review of this work, see Krishna (2009, Chap. 4). In our analysis, in addition to the asymmetry over qualification, we allow cost asymmetry in two ways: the incumbent’s underlying production cost distribution can be different from the entrant’s, and the buyer can use bidder-specific markups to account for her “switching costs.”

A handful of papers have empirically studied incumbent and entrant bidding in auctions. Zhong (2007) examines incumbent and entrant behavior in multi-item procurement auctions for direct goods held by a large high-tech company. Consistent with our model and experimental findings, her empirical data suggests that incumbents seem to choose between timid testing and all-out competing for the contract, and that incumbents often win the contract without being the lowest bidder. Tunca et al. (2010) study the effects of incumbency and supplier service quality based on data from legal service procurement auctions. They use a score auction model and empirically demonstrate that the traditionally perceived incumbent bias can be explained by the buyer’s revealed preference of quality. De Silva et al. (2003) study sealed bids for road construction contracts for which all bidders are prequalified and the low bid always wins the contract. Finding that entrants with low costs bid more aggressively and win the auction with lower bids than incumbents, they explain this with an affiliated costs model in which entrants have more dispersed costs. Our model uses an independent private costs framework and predicts that entrants with all cost types bid more aggressively than incumbents, in our case because the incumbent strategically holds back on bidding.

Finally, in the operations literature, papers have studied competition between incumbent and entrant suppliers in settings different from ours. For example, Li and Debo (2009a, b) study a buyer’s strategic choice between sole and second sourcing, and between short-term and long-term strategies, when the buyer needs to invest in suppliers’ capacity.

3. Model
We consider a procurement manager, or buyer, who seeks to award a single, indivisible contract for goods or services. The buyer already has a preexisting incumbent supplier, denoted by $i$, who currently performs the contract. As is common in practice, we assume that the contract covers a finite period of time (e.g., one to two years), after which point it must be renegotiated. To this end, we assume that the buyer becomes aware of an entrant, denoted by $e$, a new supplier who approaches the buyer seeking out new business. The buyer is interested in leveraging supply-side competition for the contract by conducting an auction in which she solicits competing bids from both the incumbent and the entrant. We let $R > 0$ denote the price the incumbent currently charges the buyer for the contract; thus, the incumbent’s true cost to perform the contract, denoted by $x_i$, is assumed to be at most $R$. We assume that $x_i$ is distributed according to a cumulative distribution function $F_i$ (with probability density function $f_i$). The entrant’s true cost, $x_e$, is assumed to be a bounded random variable following a cumulative distribution $F_e$ with probability density $f$, where $F_e/f$, is increasing,1 and we assume that

1 This assumption is satisfied, for example, when $F$ is log-concave, including uniform, normal, logistic, and exponential distributions.
parameters are normalized such that zero is the left endpoint of \( x_i \)'s domain. Costs \( x_i \) and \( x_e \) are privately known and independently distributed, and the distributions \( F_i \) and \( F_e \) are common knowledge.

We assume that both suppliers seek to maximize their expected utility. We let \( U(\cdot) \) denote the incumbent’s utility function. Thus, the utility of an incumbent with true cost \( x_i \) is \( U(p-x_i) \) if he wins the contract and receives payment \( p \) from the buyer, or is \( U(0) \) if he does not win the contract. We assume that \( U(\cdot) \) is concave, i.e., the incumbent is risk neutral or risk averse. In our model setting, the entrant will have a dominant bidding strategy (see §4); thus, we do not explicitly specify the entrant’s utility function. Our theoretical analyses of the suppliers’ bidding behavior utilizes the Bayesian Nash equilibrium concept, which is standard in the auction literature.

Because of his incumbency status, the incumbent is already qualified for the contract; because of opaque bidding behavior utilizes the Bayesian Nash equilibrium we do not explicitly specify the entrant’s utility function. Thus, the buyer and the suppliers only know that the probability that the entrant is indeed qualified equals \( 0 < \beta < 1 \), the entrant’s qualification probability. The buyer’s qualification requirements (which we treat as exogenous)\(^4\) drive the size of \( \beta \). For instance, \( \beta \) close to one corresponds to very light qualification checks that any entrant supplier is very likely to pass, whereas \( \beta \) close to zero corresponds to very strict qualification requirements that relatively few entrant suppliers would be able to pass. Qualification screening checks can be costly, involving tests of supplier products, trips to the supplier’s production facilities, etc. We let \( K \geq 0 \) denote the qualification cost, that is, the cost that would be incurred by the buyer to verify whether the entrant is, or is not, qualified for the contract. We focus on the case where the buyer bears all qualification expenses, but our results easily extend to cases where the entrant also bears qualification costs; see §7.

The buyer seeks to minimize her expected total procurement cost, that is, the contract price plus any supplier qualification costs. If the buyer holds an auction in which the entrant and incumbent compete, bidding occurs via an open-descending, price-only auction as follows: The calling price begins at an initial price, and then falls continuously until at least one of the two bidders drops out. (Open-descending price-only auctions with a continuously falling price are also known as “reverse clock auctions”; see Ausubel and Cramton 2006 for discussions about clock auctions in practice.) The buyer faces a choice about the timing of qualification screening on the entrant: she can choose either “postqualification” or “prequalification,” as described next.

### 3.1. Postqualification

Under postqualification, the buyer directly conducts an open-descending, price-only auction between the entrant and incumbent, but without qualifying the entrant ahead of time. For simplicity, we assume that the auction kicks off with a calling price \( p \) equal to \( R \) (the reserve price), and the calling price \( p \) continuously drops as the auction progresses. The auction ends when either or both bidders drop out (ties are broken randomly). Suppose the auction ends at a calling price \( p = b \). If it was the entrant that dropped out first, the incumbent wins the contract and gets paid \( b \). Otherwise, the buyer performs qualification screening on the entrant and awards the contract to the entrant with a payment \( b - K/\beta \) if the entrant passes, but contracts with the incumbent and pays the incumbent \( b \) if the entrant fails.\(^5\)

Note that the buyer would be indifferent between postqualifying a price offer of \( b - K/\beta \) from the entrant and directly accepting a price offer of \( b \) from the incumbent, because \( (b-K/\beta) + b(1 - \beta) + K = b \). By subtracting \( K/\beta \) when computing the entrant’s contract payment, the buyer is ex post rational in accounting for the need to postqualify the entrant. One can regard \( K/\beta \) as a markup reflecting “switching

\(^2\)Some aspects of qualification are straightforward, such as having three satisfied references or passing product conformance tests, but others are more ambiguous. For instance, the buyer might have their internal production and design engineers conduct face-to-face meetings with the supplier’s employees, to assess the “fit” between the buyer and supplier organizations on important issues such as lean principles and quality assurance, or to verify the supplier’s in-house technical expertise on the product they are to produce (see Wan and Beil 2009).

\(^3\)For simplicity, we assume that the qualification probability and supplier cost are independent. In reality, some qualification requirements might be associated with higher cost (such as requiring the supplier to hold surge capacity), whereas others could be associated with lower cost (such as requiring preventative maintenance to reduce unanticipated downtime). The independence assumption is most appropriate in cases where the list of qualification requirements does not tend to push overall costs higher or lower.

\(^4\)The strictness of the qualification screening performed by the buyer is typically commensurate with the buyer’s perceived downside risk of supplier nonperformance. Whereas the buyer might be satisfied with only light qualification screening when purchasing indirect goods or services (such as office cleaning), when purchasing a direct input whose conformance to precise design specifications directly impacts the safety or performance of the buyer’s product, the buyer’s screening checks could be quite involved.

\(^5\)The buyer can reasonably implement such a payment rule when she can commit to ignore renegotiation offers from suppliers after the auction. In other words, suppliers know their bids will be treated as binding. Given the auction rules, the suppliers plan accordingly.
costs” related to the need to perform costly qualification screening on the entrant.\(^6\) Intuitively, as the cost of qualification \(K\) increases or the entrant’s qualification probability \((\beta)\) decreases, the entrant becomes less attractive to the buyer, which is reflected by a larger markup \(K/\beta\). In effect, the markup shifts the distribution of the entrant’s effective cost (i.e., true cost plus the markup) to the right, making the entrant less competitive. Of course, additional switching costs that are unrelated to qualification—such as the need to change order-processing procedures—could also be incorporated into the model by simply shifting \(F_x\) to the right. We assume \(R > K/\beta\), otherwise, no entrant cost type could earn a positive profit.

3.2. Prequalification

Under prequalification, the buyer pays \(K\) to screen the entrant before the auction. With probability \(1 - \beta\), the entrant is found to be unqualified and is discarded, and without any competitive threat to the incumbent, the contract is de facto renewed with the incumbent at prevailing price \(R\). This captures situations where the buyer can only gain price concessions through supplier competition. However, with probability \(\beta\), the prequalification establishes that the entrant is qualified, at which point an open-descending price-only auction is conducted between the entrant and the incumbent. In the auction, whichever bidder drops out first loses (ties are broken randomly), and the other wins the contract and is paid the loser’s dropout bid.

4. Theoretical Analyses

4.1. Incumbent Bidding Strategy

Under Postqualification

Standard open-descending auction analyses conclude that a supplier who must win the auction to win the contract finds it a dominant strategy to stay in the auction until either their opponent drops out or the price drops so low that winning would be unprofitable, whichever happens first (for the forward auction analogue, see the discussion of English auctions in Krishna 2009). The implication is that if the buyer chooses prequalification and the entrant passes prequalification, both suppliers would bid down to their true cost before dropping out in the ensuing auction. It also implies that under postqualification, the entrant finds it optimal to bid down to his true effective cost \((x_e + K/\beta)\). However, the standard theory does not specify how the incumbent should bid under postqualification, because then the incumbent can lose the auction but still win the contract. The following theorem characterizes the incumbent’s Bayesian Nash equilibrium bidding strategy under postqualification, addressing Research Question 1.

**Theorem 1.** Given any \(F_x, F_q\), \(U(\cdot)\), \(\beta \in (0, 1)\), and \(R\), for an incumbent with cost \(x_c\), there exists a static optimal bid-down-to level \(p(x_c)\) such that the incumbent should stay in the auction until the price reaches \(p(x_c)\) or the entrant drops out, whichever happens first. There exist two thresholds \(x_b\) and \(x_w\), such that \(x_w \leq x_b\), \(x_b < R\), \(x_b \leq K/\beta\), and

(i) \(p(x_e) = R\) if and only if \(x_e \geq x_b\);
(ii) \(p(x_w) = K/\beta\) if and only if \(x_w \leq x_b\);
(iii) \(x_b < p(x_e)\), \(K/\beta < p(x_w) < R\), and \(p(x_e)\) strictly increases in \(x_e\) if \(x_w < x_e < x_b\).

In words, Theorem 1 predicts that, depending on his true cost, \(x_e\), the incumbent deploys one of three types of strategies: “boycott the auction,” “bid to win,” and “test the water.” Under the boycott strategy, the incumbent drops out of the auction at the reserve price and simply hopes that the entrant fails postqualification. Part (i) predicts that this strategy is used when the incumbent knows he is unlikely to beat the entrant on price alone, i.e., when the incumbent’s cost is quite high \((x_e \geq x_b)\). In such a case the incumbent thinks it will likely be pointless to try and win on price, so he short-circuits the auction by dropping out immediately. Part (ii) predicts that only when his cost is very low \((x_w < x_b)\) will the incumbent deploy the bid-to-win strategy in which he stays in the auction until the entrant is forced to drop out. In fact, because \(x_w\) is bounded by the lower bound on the entrant’s effective cost distribution \((K/\beta)\), the incumbent only bids this aggressively when he is absolutely certain he can beat the entrant on price alone, i.e., when the incumbent’s cost is quite high \((x_e \geq x_b)\). Thus, in such a case the incumbent will abandon the effort and drop out at price \(p(x_e)\) before reaching his true cost. Surprisingly, this structure of the incumbent’s bidding strategy (Theorem 1) is predicted regardless of the particular entrant and incumbent cost distributions \(F_x\) and \(F_q\) and the incumbent’s utility function \(U\) (because the entrant has a dominant strategy, the result also applies regardless of the entrant’s utility function).

The key insight here is that buyers should not assume that her incumbent suppliers will bid aggressively to retain the contract just because she pits them against an entrant in an auction. The next result predicts how the incumbent’s bidding behavior changes...
with the underlying procurement setting, addressing Research Question 2.

**Theorem 2.** The optimal bid-down-to level \( p(x_i) \) increases in \( R \) for all \( x_i \). Furthermore, it decreases in \( K \) for all \( x_i \) such that \( p(x_i) > K/\beta \), and \( p(x_i)|_{K=\hat{K}} = \hat{K}/\beta \) implies \( p(x_i)|_{K=\tilde{K}} = \tilde{K}/\beta \) for all \( K > \hat{K} \). Thus, the probability that the incumbent wins the auction outright (i.e., \( p(x_i) < x_e + K/\beta \)) increases in \( K \) and decreases in \( R \). However, \( p(x_i) \) and the probability that the incumbent wins the auction outright are generally not monotone in \( \beta \).

Theorem 2 predicts that as it becomes costlier for the buyer to perform qualification screening on the entrant (as \( K \) increases), the incumbent is more likely to win the auction outright. This is because a higher qualification cost causes the buyer to add a higher “switching cost” \( (K/\beta) \) to the entrant’s bid, making it easier for the incumbent to beat the entrant on price and thereby encouraging the incumbent to try to win the auction. However, Theorem 2 predicts that the incumbent is more likely to drop out of the auction early when the reserve price \( R \) is large. By dropping out early and letting the entrant win the auction, the incumbent loses the contract if the entrant survives postqualification. This risk is more worthwhile for the incumbent if there is more profit that he is trying to preserve. Thus, the theory predicts that high profit potential for the incumbent may lead him to short-circuit the competition rather than compete harder to retain the contract.

Theorem 2 also predicts that the incumbent may bid more or less aggressively in response to a higher probability \( (\beta) \) that the entrant would survive qualification screening. As one might intuitively expect, the incumbent is encouraged to try to win the auction when the entrant is more likely to survive postqualification. However, what is perhaps less initially obvious is that the incumbent may also be discouraged from trying to win against an entrant who is more likely to survive the postqualification; this can happen because the “switching cost” shrinks as \( \beta \) increases, scaring the incumbent away from trying to compete on price. In other words, the theory predicts that when the incumbent can lose the auction but win the contract, a tougher competitor—an entrant who is more likely to be qualified—might actually forestall competition.

### 4.2. Buyer’s Optimal Qualification Strategy

In the previous section, we saw that the buyer should be careful not to assume that suppliers will bid aggressively simply because she pits them against each other in an auction. Prequalification is a tool available to the buyer to try and ensure that suppliers do bid aggressively. However, it is not clear whether the buyer would always prefer to use prequalification, which can backfire if the entrant fails prequalification and must be discarded. The buyer’s decision depends on how the incumbent will bid in an auction with postqualification. If the buyer thinks the incumbent will bid very aggressively even if the entrant might be unqualified, postqualification can be an attractive strategy. On the other hand, if the incumbent will only bid aggressively if the buyer can tout the fact that the entrant is fully qualified and only the low bid will win the contract, the buyer may be forced to use prequalification. In this section, we address the buyer’s choice between pre- and postqualification, addressing Research Question 3.

As discussed in §3, under the prequalification strategy, the buyer spends qualification cost \( K \) on qualifying the entrant, which yields one of two outcomes: With probability \( \beta \) the entrant is found to be qualified and an auction is subsequently run with two fully qualified bidders, thus resulting in an expected contract payment of \( E \max\{\min\{x_e, R\}, x_i\} \) (recall that \( x_e \leq R \)); with probability \( 1 - \beta \) the entrant is found to be unqualified and is discarded, and consequently, the contract is renewed with the incumbent at price \( R \). In summary, under prequalification the buyer’s expected total (payment plus qualification) cost is

\[
\beta E \max\{\min\{x_e, R\}, x_i\} + (1 - \beta) R + K. \tag{1}
\]

Per Theorem 1, if the buyer uses the postqualification strategy, the incumbent’s bidding strategy can be described as a bid-down-to level \( p(x_i) \). The entrant wins the auction if \( p(x_i) > \min\{x_e + K/\beta, R\} \); if so, the buyer incurs a qualification cost \( K \) to vet the entrant and pays \( p(x_i) - K/\beta \) to the entrant if the entrant survives postqualification (which happens with probability \( \beta \)), but pays \( p(x_i) \) to the incumbent if the entrant fails postqualification (which happens with probability \( 1 - \beta \)). Otherwise, if \( p(x_i) \leq \min\{x_e + K/\beta, R\} \), the incumbent wins the auction and thus keeps the contract with a payment from the buyer equal to either the entrant’s dropout bid or the reserve price (whichever is smaller), \( \min\{x_e + K/\beta, R\} \). Therefore, under the postqualification strategy, the buyer’s expected total cost is

\[
E \max\left\{ \min\{x_e + \frac{K}{\beta}, R\}, K \right\} + \beta \left[ p(x_i) - \frac{K}{\beta} \right] + (1 - \beta) p(x_i) \right\},
\]

\[
= E \max\left\{ \min\{x_e + \frac{K}{\beta}, R\}, p(x_i) \right\}. \tag{2}
\]

The buyer finds the optimal qualification strategy by comparing (1) with (2). The next proposition proves the buyer prefers postqualification screening if the qualification cost \( K \) is large enough.
Theorem 3. Given any $F_i$, $F_e$, $U(\cdot)$, $\beta \in (0, 1)$, and $R$, there exists a threshold $K$ such that it is optimal for the buyer to choose postqualification if $K > K$. In particular, $K$ approaches zero whenever $\beta$ approaches zero from above or $\beta$ approaches one from below.

When the qualification cost is high enough, postqualification screening is preferred because it helps the buyer avoid wasting money qualifying an entrant whose price in the auction might not turn out to be competitive. Although Theorem 3 predicts that the buyer’s optimal strategy can be characterized by a threshold over qualification cost, it also indicates that there in general does not exist a similar threshold over qualification probability $\beta$: A buyer can prefer postqualification either when $\beta$ is small enough (in which case prequalification is very likely to disqualify the bidder) or large enough (in which case the incumbent is inclined to bid aggressively even under postqualification because he knows the entrant would stand only a small chance of failing postqualification). Moreover, one can show that replacing $p(x_i)$ with $x_i$ in Equation (2) always yields a cost smaller than Equation (1) (we omit the algebra showing this), meaning that the buyer would always prefer postqualification if the incumbent never held back on bidding. Thus, as described at the outset of this subsection, the buyer’s decision depends on how the incumbent will bid in an auction with postqualification.

5. Experimental Design and Research Hypotheses

5.1. Experimental Design

We designed our experiments to test the research hypotheses in a way that gives the theory the best shot to work by simplifying the participants’ decision tasks and promoting learning. With this goal in mind, we start by analyzing the behavior of incumbent suppliers and buyers separately.

In all our laboratory settings, an incumbent supplier competes with an entrant for the right to provide a contract to the buyer. The entrant has a dominant strategy to bid truthfully, so we automated the entrant to bid according to this dominant strategy in all experiments. The incumbent’s cost of providing the contract is $U[10, 110]$ and the entrant’s cost is $U[0, 100]$. Both costs are rounded to the nearest integer. In our treatments we vary two factors at two levels. The first factor is $K$, ranging from 1–50, and the second factor is $\beta$, with either high (70%) or low (30%). In all treatments, we set the reserve $R = 110$ at the top of the incumbent’s cost distribution support. Under postauction qualification screening, the entrant’s effective cost is increased by $K/\beta$, making the effective cost distribution $U[K/\beta, K/\beta + 100]$.

As is common in the experimental auctions literature, our theoretical predictions of incumbent bidding are derived under the assumption of risk neutrality. Figure 1 shows the incumbent’s optimal bid function (bid-down-to level) with postauction qualification screening in the four $K/\beta$ combinations of our experiments.7 As predicted by Theorem 1, when incumbent suppliers have to bid with postauction qualification screening, their problem is especially complex: As can be seen from Figure 1, the bidding strategy can be radically different depending on the incumbent’s cost $x_i$. Figure 1 reveals, generally speaking, the following trend: When $x_i$ is high enough, incumbents should boycott the auction by bidding $b = R_i$; when $x_i$ is low enough, incumbents should bid to win; and when $x_i$ is at intermediate levels, incumbents should bid above their cost.

We tested the incumbent’s behavior in a set of treatments called the Incumbent experiment. We kept the same $\beta$ for the entire session and varied $K$ after 50 rounds (half of the subjects had $K = 2$ in rounds 1–50 followed by $K = 20$ in rounds 51–100, and the

7 Figure 1 is based on closed-form expressions of the optimal bid function, derived in the appendix.
order was reversed for the other half of the subjects), but in each round the incumbent received a new independent random draw of \( x_i \sim U[10, 110] \) and was allowed to place any integer bid between \( x_i \) and \( R = 110 \).

We tested how human buyers select the timing of the auction qualification screening when faced with an automated incumbent and entrant, both programmed to bid optimally, in a set of treatments called the Buyer experiment. In this experiment human subjects were in the buyer role and completed 50 rounds. Buyers received a fixed revenue each round, which we set to be \( 120 + K \). Computerized suppliers received new independently drawn costs each round, \( x_e \sim U[10, 110] \) for the incumbent and \( x_e \sim U[0, 100] \) for the entrant. The buyer had to decide on the pre- or postauction qualification screening at the beginning of the round. If the buyer chose the preauction qualification screening, the entrant was either successfully qualified (with probability \( \beta \)) or not (with probability \( 1 - \beta \)). If the entrant failed the screening, the round ended with the contract price of 110 and the buyer earned \( 10(=120+K-K-110) \). If the entrant passed the screening, an auction was conducted in which the losing supplier bid down to his cost, the auction ended at price \( P = \min(\max(x_i, x_e), R) \), and the buyer earned \( 120 + K - K - P = 120 - P \). If the buyer chose the postqualification screening, the auction was conducted immediately, and because the incumbent was programmed to bid down to \( p(x_i) \), the auction ended at price \( P = \min(\max(x_i, K/\beta, p(x_i)), R) \). At this point, if the incumbent won, the round ended and the buyer earned \( 120 + K - P \). If the entrant won, however, the qualification screening was conducted. If the entrant survived postqualification (with probability \( \beta \)), he won the contract and was paid \( P \). If the entrant failed postqualification (with probability \( 1 - \beta \)), the incubent won the contract and was paid \( P \). The buyer’s average cost under postqualification was thus \( 120 + K - P \). We varied the \( \beta \) and \( K \) parameters between subjects, so each participant faced only a single \( \beta \) and \( K \) combination.

The third and last experiment (called the Buyer–Incumbent experiment) involved human players in both roles (the buyer and the incumbent supplier). Each participant kept his or her role throughout the session and was randomly matched each round with another human participant with a different role. Incumbents’ costs were randomly drawn each round according to \( x_i \sim U[10, 110] \), and they could place any integer bid between \( x_i \) and \( R \).

Figure 2 shows the theoretical predictions for the buyer’s optimal decision for various qualification cost and probability pairs, \((K, \beta)\). The hill-shaped line divides the plane: In the upper region the buyer finds it optimal to choose the postqualification strategy, and in the lower region the buyer finds it optimal to use the prequalification strategy. Confirming Theorem 3, the theory predicts that the buyer prefers prequalification only when the qualification cost \( K \) is relatively small and the size of the qualification probability \( \beta \) has a nonmonotone effect on the buyer’s decision: The buyer prefers prequalification when \( \beta \) is neither too small nor too large.

In the Incumbent experiment, 21 participants were in the \( \beta = 0.3 \) treatment and 22 participants were in the \( \beta = 0.7 \) treatment (for a total of 43). In the Buyer experiment, 19 participants were in the \( K = 2|\beta = 0.3 \) treatment, 19 participants were in the \( K = 20|\beta = 0.3 \) treatment, 12 participants were in the \( K = 2|\beta = 0.7 \) treatment, 10 participants were in the \( K = 20|\beta = 0.7 \) treatment (for a total of 60), and 28 participants were in the Buyer–Incumbent treatment (14 buyers and 14 incumbents). In total, 131 participants were included in our study.

We conducted all experimental sessions at the Laboratory for Economic Management and Auctions at the Smeal College of Business, Pennsylvania State University. Our participants were students, mostly undergraduates, from a variety of majors. We recruited them through the online recruitment system, offering earning cash as the only incentive to participate. Upon arrival at the laboratory, the subjects were seated at computer terminals. We handed out written instructions (see the online appendix, available at http://www.utdallas.edu/~emk120030/SampleInstructions.pdf, for samples) to participants. After they read the instructions, to ensure common knowledge about the game’s rules, we then read the instructions aloud before starting the actual game. No further decision support was provided. Thus, as is often the case in practice, subjects in our experiments relied on
human judgement when strategizing how much to bid (as opposed to, say, relying on sophisticated decision support tools).

In each session, each participant completed a number of rounds. Each individual always had the same role (buyer or supplier) in each round. We programmed the experimental interface using the zTree system (Fischbacher 2007). At the end of each session we computed cash earnings for each participant by multiplying the total earnings from all rounds by a predetermined exchange rate and adding it to a $5 participation fee. Participants were paid their earnings in private and in cash, at the end of the session.

5.2. Theoretical Benchmarks and Research Hypotheses

As we can see from Figure 1, incumbent bid functions are quite complex, making it difficult to compare bidding behavior across treatments. Therefore, we establish several metrics that allow us to compare behavior across combinations of \( K \) and \( \beta \) as well as to test whether behavior is qualitatively consistent with theoretical benchmarks. The metrics we use are (1) boycott rates: proportion of \( b = R \) bids; (2) bid-to-win rates: proportion of \( b \leq \max(x_i, K/\beta) \) bids; (3) winning rates: proportion of auctions the incumbent wins outright; and (4) contract prices: the average auction price \( \bar{P} \).

We formulate our first hypothesis to measure the extent to which the bids in the Incumbent treatment match theoretical predictions. We could formulate the strong version of the hypothesis (predicting that subjects will precisely follow the theoretical threshold strategy), but because human behavior tends to be noisy, it is unlikely that human subjects will literally follow the threshold bidding strategy. Instead, we formulate a weaker version of the hypothesis to check the relationship between the actual and optimal bids. The following research hypothesis is based on Figure 1.

Hypthesis 1A. In the Incumbent treatments, boycott rates should increase in \( x_i \) and the bid-to-win rates should decrease in \( x_i \) in all treatments except for two cases: in the treatment with \( K = 2 \) and \( \beta = 0.3 \), the boycott rates and the bid-to-win rates should be independent of \( x_i \); in the treatment with \( K = 2 \) and \( \beta = 0.7 \), the bid-to-win rates should be independent of \( x_i \).

In the Incumbent treatments, the boycott and bid-to-win rates may not be very meaningful, because the number of possible bids incumbents can place is large. An alternative to \( b = R \) or \( b \leq \max(x_i, K/\beta) \) may be any bid between \( \max(x_i, K/\beta) \) and \( R \). If the optimal bid is \( R \) and the actual bid placed is slightly below \( K \), for example, this deviation is likely to have a negligible effect on the auction outcome in terms of who wins and the resulting contract price. Therefore, when comparing different conditions in the Incumbent treatments we will also focus on metrics (3) and (4), the proportion of auctions incumbents win outright and the resulting average contract prices, respectively. Table 1 shows optimal incumbent winning rates and contract prices in square brackets at the top of each cell, for the Incumbent treatments. These optimal average contract prices and winning rates are based on the actual realizations of \( x_i \) and \( x_c \) in the experiment.

Hypthesis 1B. In the Incumbent treatments, average incumbent winning rates and contract prices should not be different from the benchmarks in Table 1.

The next set of hypotheses follow from the predictions of Figure 1 and Theorem 2 (as captured in the benchmarks in Table 1), and they deal with effects of \( K \) and \( \beta \) on boycott rates, winning rates, and contract prices in the Incumbent treatments.

<table>
<thead>
<tr>
<th>( H_i ); the effect of ( K )</th>
<th>( \beta = 0.3 )</th>
<th>( \beta = 0.7 )</th>
<th>( H_i ); the effect of ( \beta )</th>
<th>( \beta = 0.3 )</th>
<th>( \beta = 0.7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 2 )</td>
<td>( [0.000] )</td>
<td>( [0.200] )</td>
<td>( p &lt; 0.0001 )</td>
<td>( [107.97] )</td>
<td>( [94.94] )</td>
</tr>
<tr>
<td>( 0.096^{**} )</td>
<td>( 0.298^{**} )</td>
<td>( (0.103) )</td>
<td>( (0.143) )</td>
<td>( 98.75^{**} )</td>
<td>( 84.17^{**} )</td>
</tr>
<tr>
<td>( K = 20 )</td>
<td>( [0.667] )</td>
<td>( [0.525] )</td>
<td>( p = 0.0591 )</td>
<td>( [101.18] )</td>
<td>( [91.42] )</td>
</tr>
<tr>
<td>( 0.670 )</td>
<td>( 0.555 )</td>
<td>( (0.148) )</td>
<td>( (0.159) )</td>
<td>( 100.45^{*} )</td>
<td>( 91.44 )</td>
</tr>
</tbody>
</table>
| \( H_i \); the effect of \( K \) | \( p < 0.0001 \) | \( p < 0.0001 \) | \( p = 0.8501 \) | \( p = 0.0039 \)

Note. \( H_i \); Realized amount = theoretical amount is rejected. \(^{*}p < 0.1; ^{**}p < 0.01.\)
Hypothesis 2A. Average incumbent winning rates in the Incumbent treatments should increase as $K$ increases, which implies that incumbents will win more often when $K = 20$ than when $K = 2$ for both levels of $\beta$.

Hypothesis 2B. Sometimes higher $\beta$ causes incumbents to win more auctions outright, and sometimes fewer. When $K = 2$, incumbents should win more auctions when $\beta = 0.7$ than when $\beta = 0.3$. However, when $K = 20$ the relationship is reversed, and incumbents should win more auctions when $\beta = 0.3$ than when $\beta = 0.7$.

Hypothesis 2C. Average contract prices should decrease in $K$ and $\beta$.

The last set of benchmarks and hypotheses deals with the buyer’s behavior. In both of our buyer experiments the main metric of interest is the proportion of postauction qualification screening decisions made. Figure 2 illustrates the buyer’s optimal decisions. A secondary metric is the average total cost incurred by the buyer (contract payment plus the cost of qualification screening). Theorem 3 shows that generally, independent of $\beta$, there is a threshold in $K$ such that when $K$ is high enough, the buyer is better off always using postauction qualification screening.

Hypothesis 3A. Buyers should select preauction qualification screening when $K = 2$ regardless of the $\beta$, and postauction qualification screening when $K = 20$, also regardless of the $\beta$.

Hypothesis 3B. This behavior should not be affected by whether the incumbent suppliers are human subjects or automated agents programmed to follow the optimal bidding strategy.

6. Experiment Results

6.1. Do the Theory’s Point Predictions Match the Data?

We begin testing Hypothesis 1A by looking at the incumbents’ bidding behavior. The top panels of Figure 3 present scatterplots of incumbents’ bids as a function of $x_i$ whenever those bids were between $\max(x_i, K/\beta)$ and $R$. The bottom panels of Figure 3 present the proportion of $b = R$ and $b \leq \max(x_i, K/\beta)$ bids as a function of $x_i$. Figure 3 reveals that participants often employed intermediate bidding strategies. This experimental evidence suggests that intermediate strategies can be expected in practice, despite being more complicated than extreme strategies.

There is no evidence in Figure 3 that incumbents use optimal threshold strategies, so we focus on qualitative predictions. To test Hypothesis 1A, we estimated two logit models (with random effects\footnote{Random-effects models are used to address individual heterogeneity in all our regression results. We also did the following as each treatment. The first is the model of boycotting, with the dependent variable that is 1 when $b = R$, and the second is the model of bidding to win, with the dependent variable that is 1 when $b \leq \max(x_i, K/\beta)$. Independent variables are $x_i$ and $\text{Round}$ (to control for learning). We present results in Table 2.

The coefficients of $x_i$ are positive and significant in all four boycotting models, so although incumbents do not follow the threshold strategy for boycotting, they are more likely to boycott when their costs are high, which is at least qualitatively consistent with the optimal bidding behavior and Hypothesis 1A. The one exception is the $K = 2, \beta = 0.3$ treatment, for which the hypothesis predicted no effect of $x_i$ on the boycotting rate. In this case, boycotting was optimal for all cost values, and indeed, incumbents seemed more inclined to boycott in this treatment than in the other treatments, as evidenced by the relatively high proportion (ranging from 40% to 80%) of boycotting bids (Figure 3(c)’s lower panel). In the models of bidding to win, the coefficients of $x_i$ are negative and significant in the two $K = 20$ treatments and the $x_i$ coefficient is not significant in the $K = 2, \beta = 0.7$ treatment—all consistent with Hypothesis 1A. In the $K = 2, \beta = 0.3$ treatment, however, the $x_i$ coefficient is positive and significant, which at first glance appears to be a serious contradiction. However, the total proportion of these low bids is less than 1% in the $K = 2, \beta = 0.3$ treatment (also refer to Figure 3(c)’s lower panel), so the deviation is not serious in absolute terms.

To analyze bids between $R$ and $\max(x_i, K/\beta)$, we estimate a tobit model for each treatment (recall that the bids are restricted to be between $R$ and $x_i$) in which we regress the bid $b$ on the optimal bid $b^*$, as well as the $\text{Round}$ to control for learning. We estimate these tobit models with random effects and present results in Table 3. In the two $\beta = 0.7$ treatments, the $\text{Constant}$ is not significantly different from 0, and the $b^*$ coefficient is significantly lower than 1 ($p < 0.05$ for both treatments), indicating that on average bids are lower than the optimal bids. The $\text{Round}$ coefficient is not significant in the $K = 20, \beta = 0.7$ treatment, but it is positive and significant in the $K = 2, \beta = 0.7$ treatment, indicating that on average incumbents are learning to
Figure 3  Bids as a Function of $x_i$ in Incumbent Treatments

(a) $K = 2/\beta = 0.7$

(b) $K = 20/\beta = 0.7$

(c) $K = 2/\beta = 0.3$

(d) $K = 20/\beta = 0.3$

Note. The top panels present scatter plots of incumbents’ bids between $\max(x_i, K/\beta)$ and $R$; the bottom panels present the proportion of bids equal to $R$ and less than $\max(x_i, K/\beta)$. 
that on average, bids are in fact significantly lower
so incumbents do bid more aggressively than they
is positive and significant (and large), the
Constant model in the
appears that on average bids are consistent with our
mal bid is always
round). In the
bid slightly higher over time (about 0.13 ECU per
round). In the \( K = 2 \mid \beta = 0.3 \) treatment, the optimal bid is always \( b^* = R \), so the \( b^* \) coefficient cannot be estimated. The \( \text{Constant} \) is not significantly different from \( R = 110 \), and the \( \text{Round} \) coefficient is also positive and significant, so once we account for random effects and for the data censoring, it appears that on average bids are consistent with our model in the \( K = 2 \mid \beta = 0.3 \) treatment. Note, however, that on average, bids are in fact significantly lower than \( R \) in that treatment (refer back to Figure 3(c)), so incumbents do bid more aggressively than they should, but our tobit model indicates that we cannot reject the hypothesis that the deviations can be due merely to noise. In the \( K = 20 \mid \beta = 0.3 \) treatment, the \( \text{Constant} \) is positive and significant (and large), the
\( b^* \) coefficient is much lower than 1, and the \( \text{Round} \) coefficient is not significant, so in that treatment incumbents bid too high, on average, when their costs are low, but bid too low when their costs are high.

Overall, incumbents bid more aggressively than they should because they do not boycott enough auctions, and when they do not follow one of the two extreme strategies, their bids are generally lower than they should be. We now analyze the effect of this bidding behavior on auction outcomes, addressing Hypothesis 1B. Table 1 presents information about incumbent winning rates and contract prices. Contract prices in Table 1 give a sense of what the winning suppliers are paid on average. We first compare winning rates and contract prices to their theoretical benchmarks, presented in square brackets at the top

<table>
<thead>
<tr>
<th>Treatment: ( K = 2 \mid \beta = 0.7 )</th>
<th>( K = 20 \mid \beta = 0.7 )</th>
<th>( K = 2 \mid \beta = 0.3 )</th>
<th>( K = 20 \mid \beta = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b = R )</td>
<td>( b \leq \max(x_i, K/\beta) )</td>
<td>( b = R )</td>
<td>( b \leq \max(x_i, K/\beta) )</td>
</tr>
<tr>
<td>( x_i )</td>
<td>0.086**</td>
<td>0.005</td>
<td>0.089**</td>
</tr>
<tr>
<td>(0.0050)</td>
<td>(0.003)</td>
<td>(0.008)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>( \text{Round} )</td>
<td>0.051**</td>
<td>0.036**</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>( \text{Constant} )</td>
<td>-8.36**</td>
<td>-4.26**</td>
<td>-9.03**</td>
</tr>
<tr>
<td>(0.85)</td>
<td>(0.84)</td>
<td>(1.05)</td>
<td>(0.70)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-364.71</td>
<td>-368.93</td>
<td>-278.22</td>
</tr>
<tr>
<td>No. of obs. (groups)</td>
<td>1,100 (22)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \text{Note.} \) The group variable is individual subject. \( ^* p \leq 0.05; \quad ^{**} p \leq 0.01. \)

Table 3 Estimates of the Tobit Models of Bids Between the Two Extremes in the Incumbent Treatments

<table>
<thead>
<tr>
<th>Treatment: ( K = 2 \mid \beta = 0.7 )</th>
<th>( K = 20 \mid \beta = 0.7 )</th>
<th>( K = 2 \mid \beta = 0.3 )</th>
<th>( K = 20 \mid \beta = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^* )</td>
<td>0.703**</td>
<td>0.942**</td>
<td>N/A</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.020)</td>
<td>(0.035)</td>
<td></td>
</tr>
<tr>
<td>( \text{Round} )</td>
<td>0.129*</td>
<td>-0.063</td>
<td>0.623**</td>
</tr>
<tr>
<td>(0.066)</td>
<td>(0.043)</td>
<td>(0.080)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>( \text{Constant} )</td>
<td>9.70</td>
<td>4.44</td>
<td>100.97**</td>
</tr>
<tr>
<td>(7.56)</td>
<td>(7.39)</td>
<td>(9.39)</td>
<td>(12.99)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-3,404.45</td>
<td>-2,919.67</td>
<td>-2,147.99</td>
</tr>
<tr>
<td>No. of obs. (groups)</td>
<td>1,100 (22)</td>
<td>1,050 (21)</td>
<td></td>
</tr>
</tbody>
</table>

\( \text{Note.} \) The group variable is individual subject. \( ^* p \leq 0.05; \quad ^{**} p \leq 0.01. \)
of each cell of Table 1. In the $K = 2$ condition, winning rates are higher than they should be in theory, and in the $K = 20$ condition, winning rates are not different from their theoretical benchmarks. Contract prices are significantly lower than the theory predicts, with the exception of the $K = 20$ treatment. These conclusions are consistent with our earlier observation that bids are more aggressive than they should be.

6.2. Do the Theory’s Qualitative Predictions Match the Data?

Table 1 presents results of hypotheses tests pertinent to Hypotheses 2A and 2B; we find support for both.\(^9\) Consistent with predictions of Hypothesis 2A, incumbent winning rates are higher when $K = 20$ than when $K = 2$ for both levels of $\beta$. Consistent with predictions of Hypothesis 2B, the incumbent’s winning rates are higher when $\beta = 0.7$ than when $\beta = 0.3$ in the $K = 2$ condition. Also, consistent with Hypothesis 2B, this relationship is reversed in the $K = 20$ condition, where incumbent winning rates are significantly lower when $\beta = 0.7$ than when $\beta = 0.3$.

Turning to Hypothesis 2C, contract prices are significantly higher when $\beta = 0.3$ than when $\beta = 0.7$ for both levels of $K$, which is consistent with Hypothesis 2C. However, contrary to the hypothesis, average contract prices are not higher when $K = 2$ than when $K = 20$. In fact, when $\beta = 0.7$, average contract prices are significantly lower when $K = 2$ than when $K = 20$. Average contract prices are in line with theoretical prediction when $K = 20$, but they are significantly lower than they should be when $K = 2$. This is largely because incumbents boycott auctions much more than they are when the $K = 2$ condition.

6.3. Do Buyers Behave as Predicted When Facing Automated Incumbents?

We present the analysis related to Hypothesis 3A by fitting a logit model to the data in the Buyer treatments, with the dependent variable that is 1 when the preauction qualification screening is selected and 0 when the postauction qualification screening is selected. The independent variables are listed in Table 4; $HighK = 1$ in the $K = 20$ condition and 0 otherwise, $High\beta = 1$ in the $\beta = 0.7$ condition and 0 otherwise, and $Round$ is simply the decision number 1 to 50.

Hypothesis 3A says that participants should select preauction qualification screening when $K = 2$, and postauction qualification screening when $K = 20$, regardless of $\beta$. This means that the coefficient for $HighK$ should be negative and significant, and the coefficient for $High\beta$ should not be significant. Indeed, this is what we see in Model (1) of Table 4.

\(^9\) For all one-sample tests, we used Wilcoxon matched pairs signed-rank tests. For all two-sample tests, we used Wilcoxon rank-sum tests.

### Table 4 Logit Estimates of the Buyer Behavior; the Dependent Variable is 1 if Preauction Qualification Screening Was Selected and 0 if Postauction Qualification Screening Was Selected

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.946**</td>
<td>0.030</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td>(0.300)</td>
<td>(0.334)</td>
<td>(0.356)</td>
</tr>
<tr>
<td>$HighK$</td>
<td>$-2.674^∗$</td>
<td>$-1.028^∗$</td>
<td>$-1.717^∗$</td>
</tr>
<tr>
<td></td>
<td>(0.381)</td>
<td>(0.424)</td>
<td>(0.516)</td>
</tr>
<tr>
<td>$High\beta$</td>
<td>$-0.020$</td>
<td>0.046</td>
<td>$-0.821$</td>
</tr>
<tr>
<td></td>
<td>(0.392)</td>
<td>(0.438)</td>
<td>(0.575)</td>
</tr>
<tr>
<td>$HighK \times High\beta$</td>
<td>1.902</td>
<td></td>
<td>1.902</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.849)</td>
</tr>
<tr>
<td>$Round$</td>
<td>0.038**</td>
<td>0.036**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>$Round \times HighK$</td>
<td>$-0.070^∗$</td>
<td>$-0.064^∗$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>$Round \times High\beta$</td>
<td>$-0.012$</td>
<td>$-0.006$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>$Round \times HighK \times High\beta$</td>
<td>$-0.013$</td>
<td></td>
<td>$-0.013$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>$-1,463.42$</td>
<td>$-1,409.05$</td>
<td>$-1,406.61$</td>
</tr>
<tr>
<td>No. of obs. (groups)</td>
<td>3,000 (60)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. The group variable is individual subject. 

\(∗p < 0.05; ∗∗p < 0.01.\)

In Model (2) we add the $Round$ variable and interact it with the indicator variables for $HighK$ and $High\beta$. Again, we see that $HighK$ continues to be negative and significant, $High\beta$ continues to not be significant, and additionally, the same effect persists with experience ($Round \times HighK$ is negative and significant, and $Round \times High\beta$ is not significant). In Model (3) we capture the additional interaction effect between $HighK$ and $High\beta$. The positive and significant $HighK \times High\beta$ coefficient tells us that the effect of $K$ at the beginning of the session is stronger in the $\beta = 0.3$ than in the $\beta = 0.7$ condition, but regardless of $\beta$, buyers learn to select preauction qualification screening less often as the session progresses. The learning is not affected by the interaction effect ($Round \times HighK \times High\beta$ coefficient is not significant).

Overall, we conclude that buyers, when faced with suppliers who consistently behave in a way that adheres to our model, also learn to behave consistently with our model’s predictions; we find support for Hypothesis 3A.

6.4. Do Buyers Behave as Predicted When Facing Human Incumbents?

The Buyer–Incumbent experiment involves both human players, and we use it to test Hypothesis 3B. Participants played for multiple rounds but were in the same role, either the buyer or the incumbent supplier, for the entire session. We conducted this experiment in the $\beta = 0.7$, $K = 2$ setting only because that is the setting that was most challenging for incumbents (due to a complicated optimal bidding function) as well as buyers (due to the difficulty in responding
to the complexity that resulted from the incumbents’ complicated bidding function). We first analyze the behavior of human buyers.

When incumbents bid optimally, buyers maximize their expected profits in the \( \beta = 0.7 \mid K = 2 \) condition by using the preauction qualification screening, and we have already observed that when incumbent suppliers are automated, buyers learn, over time, to make this decision. However, in the treatments with human incumbents, the learning trend is the opposite: Over time, buyers learn to use the postauction qualification screening.

To analyze the buyer’s behavior when faced with human incumbents, we fit a logit model similar in structure to the models in Table 4. The dependent variable in the logit model is 1 if the preauction qualification screening is selected. The independent variables are as follows: the human incumbent indicator variable, which is not significant; the interaction variable between the human incumbent indication variable and \( \text{Round} \), which is negative and significant; and the interaction variable between the automated supplier and \( \text{Round} \), which is positive and significant. The data clearly rejects Hypothesis 3B—when they deal with human incumbents, buyers behave differently from the way they behave when dealing with automated incumbents.

To better understand the reasons for the different buyer behavior when faced with human versus automated incumbents, we analyze the behavior of the human incumbents. For this purpose, it is useful to examine the total cost that buyers incur. Table 5 summarizes the average total costs borne by the buyer (payment to supplier plus any qualification screening cost) under the pre- and postqualification screening strategies, for human and automated incumbents, their standard errors, and the results of hypothesis tests comparing average total costs.

The key observation is that although the buyer’s average total cost under the preauction qualification screening strategy is not significantly different for human and automated incumbents (in other words, human incumbents’ bidding behavior results in essentially the same buyer costs as the optimal incumbent behavior), the buyer’s average total cost under the postauction qualification screening is significantly lower with human incumbents than with automated incumbents (this is another piece of evidence against part of Hypothesis 1A). That is, human incumbents bid more aggressively than the optimal bidding strategy dictates. We already made the same observation when testing Hypothesis 1A in our analysis of human incumbent behavior under the postauction qualification screening when faced with automated buyers (the Incumbent treatments)—average contract prices were consistently lower than they should have been in theory. These differences in suppliers’ bidding behavior explain buyer behavior.

When incumbents are automated, preauction qualification screening results in significantly lower costs, so buyers learn to use it. When incumbents are human, preauction qualification screening results in cost that are only slightly higher than costs from the postauction qualification screening (and differences are only weakly significant), so there is no reason to expect buyers to strongly prefer one decision over the other.

### 7. Conclusions

Buyers often wish to leverage entrant suppliers against incumbent suppliers in order to reduce spend. However, incumbent suppliers might be reluctant—or even unwilling—to bid aggressively against an entrant if they suspect that the entrant is possibly unqualified. We analyze this with a two-pronged approach. First we model this phenomenon in the context of an open-descending, price-only reverse auction (which is commonly used in practice) and derive the incumbent’s optimal bidding strategy. Next, we lab test our model’s theoretical predictions. In addition to providing the first equilibrium analyses for bidding behavior in reverse open-descending, price-only auctions with possibly unqualified bidders, in testing these predictions in the lab the present paper also is the first experimental study of auctions with possibly unqualified bidders.

Addressing Research Question 1 (incumbent’s optimal bidding strategy under postqualification), we show that high- and medium-cost incumbents optimally drop out of the auction before reaching their true cost, letting the entrant win the auction; by doing so, the incumbent tries to preserve his profit margin and hopes the entrant fails postqualification. This holds under general assumptions and differs from the “bid-to-cost” strategy that would be dominant if all suppliers were fully qualified, implying that operational issues surrounding the auction (i.e., supplier qualification) can be as important for generating price concessions as the notion of an auction itself.

Addressing Research Question 2 (sensitivity of the incumbent’s bidding strategy), we find that the incumbent gains an advantage and is more likely to win the auction when it is costlier for the buyer to
perform postqualification screening on the entrant. However, the incumbent is more likely to drop out of the auction early when the reserve price is large; thus, ironically, high profit potential for the incumbent may lead him to short circuit the auction rather than compete harder to retain the contract. Our theory also predicts that the incumbent may bid more or less aggressively in response to a higher probability that the entrant would survive qualification screening, meaning that a tougher competitor—namely, an entrant that is more likely to be qualified—might actually forestall competition. Despite the fact that postqualification can cause the incumbent to hold back on bidding, if it is very expensive to qualify the entrant, the buyer prefers using postqualification, addressing Research Question 3 (buyer’s preference between pre- and postqualification).

Laboratory experiments testing our theory’s predictions (Research Question 4) reveal that, consistent with our theoretical results, postqualification does indeed cause incumbents to hold back on bidding. Another important finding is that incumbents tend to bid more aggressively than theory predicts under postqualification. Overly aggressive bidding is a well-known result in auction experiments, where it has been documented that bidders in sealed-bid first-price auctions with independent private values tend to bid above the risk-neutral Nash equilibrium (see Kagel 1995 for a thorough review). Exact reasons for the overly aggressive bidding in sealed-bid first-price auctions are not well understood. One longstanding explanation that has been offered is risk aversion (Cox et al. 1988), but several studies report results that run counter to the risk-aversion explanation (Kagel and Levin 1993, Cason 1995, Isaac and James 2000, Englebrecht-Wiggans and Katok 2009). Another explanation that seems to fit a wider range of settings is bidder aversion to regret (Feliz-Ozbay and Ozbay 2007; Englebrecht-Wiggans and Katok 2007, 2008). Although determining the exact reasons for overly aggressive bidding is beyond the scope of our paper, we note that it is tenable that we would observe such behavior in our setting, because the incumbent’s problem under the postauction qualification screening has the flavor of a sealed-bid auction, and consequently, overly aggressive bidding in our setting is likely due to the same reasons as overly aggressive bidding in sealed-bid first-price auctions.

Despite overly aggressive incumbent bidding, we find that the theory’s qualitative predictions (sensitivity of the incumbent’s bidding strategy to contract and entrant characteristics) stand up quite well in the lab—generally, most parts of Hypotheses 2A–2C are supported. The notable exception is $K$’s effect on average contract prices. Contract prices should decrease in $K$, but in our experiment they do not change for $\beta = 0.3$ and actually increase for $\beta = 0.7$. When $K$ is low, incumbents should usually boycott auction (100% of the time in the $K = 2|\beta = 0.3$ treatment and about 70% of the time in the $K = 2|\beta = 0.7$ treatment). However, our incumbents only boycott auctions 58% of the time in the $K = 2|\beta = 0.3$ treatment and 22% of the time in the $K = 2|\beta = 0.7$ treatment. Low $K$ makes entrants more competitive in the auction, because their $K/\beta$ penalty is lower. Incumbents should boycott those auctions and count on the entrant failing the postauction qualification screening rather than getting into a bidding war with them. However, this insight is far from obvious, and many of our incumbents compete with entrants aggressively instead. Interestingly, they still do not win very many auctions outright—only slightly more than they should—but they compete down the price, so even when they lose the auction the final prices are significantly lower.

A managerial implication is that this overall tendency to bid overly aggressively may make buyers in practice more willing to go ahead and pit incumbents against possibly unqualified entrants, and indeed we found evidence of this in our experiments with human buyers and human incumbents: Human buyers reacted to overly aggressive human incumbents by choosing postqualification more often than when they faced automated incumbents programmed to bid optimally.

We assumed that the buyer shoulders all the qualification expenses, but one can imagine situations where the entrant could share the burden. Suppose for $\alpha_1, \alpha_2 \in [0, 1]$, the buyer incurs $\alpha_1 K$ and the entrant incurs $(1 - \alpha_1) K$ whenever qualification screening occurs and the entrant passes the screening, and the buyer incurs $\alpha_2 K$ and the entrant incurs $(1 - \alpha_2) K$ whenever qualification screening occurs and the entrant fails the screening. Our main theoretical findings are robust to this embellishment (for brevity these results are omitted). In particular, the postqualification analysis does not change, and Theorems 1 and 2 go through as before. The intuition is that when the entrant knows he will bear qualification costs during postqualification, he inflates his bid to reflect this, and in the end the net effect of this bid price inflation acts the same as if the buyer knew she would have to bear the costs and so added a bid markup. An analogue of Theorem 3 also holds: For large enough qualification cost $K$, because the buyer bears more qualification costs ($\alpha_1, \alpha_2$ become large), the buyer eventually prefers using postqualification. In future experimental work one could test cases with various $\alpha_1, \alpha_2$. However, subjects would have to do more accounting (to keep track of how the qualification costs are divvied up), and this might lead to noisier and more error-prone decision making in the lab, which might make it more difficult to test the theory.

Our analytical and experimental analyses were enabled by stylized modeling to capture the problem’s
salient features. Several avenues for future work are possible. In this paper we examined a setting with a single entrant. However, one can imagine situations where there might be multiple entrants. With \( N \geq 2 \) entrants, as one might expect, the same key trade-offs identified in this paper still remain. For example, when facing \( N \) entrants, the incumbent holds back on bidding to the extent that he feels threatened that an entrant will bid lower and survive qualification. However, the incumbent’s bid-down-to level becomes dynamic—with \( N \) entrants the incumbent has \( N \) bid-down-to thresholds, with each threshold corresponding to the number of entrants that remain in the auction. Because of its complexity, we save a full exploration for future research.

Because we focused on the open-descending auction format, another extension would be to analyze the other format commonly used in practice, the first-price sealed-bid auction (Jap 2002). Our main insights extend to this setting: One can show that the incumbent’s best-response bid function follows a threshold structure in which low-cost incumbents bid aggressively, but higher-cost incumbents hold back on bidding or even boycott. However, solving for the equilibrium bids would be much more challenging—bidder qualification issues aside, first-price sealed-bid equilibria with asymmetric bidders have been solved for just a few very special cases (see Maskin and Riley 2000).

We took the entrant’s qualification probability, \( \beta \), as exogenous. One could incorporate our model (which predicts the buyer’s qualification plus procurement costs, given \( \beta \)) into a broader framework capturing the buyer’s trade-off between screening level (\( \beta \)) and contract nonperformance costs, with the ultimate goal of optimizing over \( \beta \). In summary, our paper reveals the importance of supplier qualification issues in industrial procurement auctions, and has the potential to help industrial buyers make better decisions about how to include entrants in procurement auctions.

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**Appendix**

**Proof of Theorem 1**

**Existence of Static Bid-Down-To Level.** During the auction, the price falls continuously, and the incumbent must dynamically decide whether to drop out. Because all actionable information revealed during the auction is subsumed by the auction price, at the auction’s outset the incumbent could decide the stay-in/drop-out decision he would make at any price. Thus, the incumbent’s optimal strategy is characterized by an optimal static bid-down-to level, \( p(x_i) \), whose existence is guaranteed because the incumbent optimizes a continuous objective function over a compact set: \( p(x_i) = \arg \max_{x_i} \left( \text{costs} \right) \), where

\[
\Pi(t) = \int_0^t \left( U(y-x_i) f_i(y-K/\beta) \right) dy + F_i \left( t-K/\beta \right) \left( 1-\beta \right) U(t-x_i) + \beta U(0) + U(R-x_i) \left( 1-F_i \left( R-K/\beta \right) \right),
\]

The decision set is \( t \in [\max(x_i, K/\beta, R] \) because the auction price can never be outside the set \( [K/\beta, R] \), and bidding below the true cost \( x_i \) can never be profitable for the incumbent. In the incumbent’s expected utility as a function of the chosen bid-down-to level \( t \), \( \Pi(t) \), the first term \( \int_0^t U(y-x_i) f_i(y-K/\beta) dy \) corresponds to the cases where the incumbent wins the auction because the entrant drops out at \( y \in (t, R) \); the second term \( F_i(t-K/\beta) \left( 1-\beta \right) U(t-x_i) + \beta U(0) \) corresponds to the cases in which the entrant wins the auction; and the last term \( U(R-x_i) \left( 1-F_i(R-K/\beta) \right) \) corresponds to the cases in which the entrant loses because his effective cost is above the reserve price.

**Existence of Thresholds \( x_0 \) and \( x_0 \).** First, we prove that \( p(x_i) \) increases in \( x_i \); next, we show the existence and uniqueness of \( x_0 \) and \( x_0 \); last, we show that \( p(x_i) \) strictly increases for \( x_i < x_i < x_0 \).  

**Monotonicity of \( p(x_i) \).** Note that

\[
\frac{d\Pi(t)}{dt} = F_i \left( t-K/\beta \right) \left( 1-\beta \right) U(t-x_i) - \beta F_i \left( t-K/\beta \right) \left( 1-\beta \right) U(t-x_i) - U(0),
\]

which strictly increases in \( x_i \) (i.e., we have \( \frac{d^2\Pi(t)}{dx_i dt} > 0 \) because \( U(t-x_i) \) increases in \( x_i \) (i.e., because \( U(\cdot) \) is concave) and \( U(t-x_i) - U(0) \) strictly decreases in \( x_i \). Because \( d\Pi(t)/dt \) strictly increases in \( x_i \), we have that \( p(x_i) \) increases in \( x_i \) and, moreover, strictly increases when \( \max(x_i, K/\beta) < p(x_i) < R \). To see this, suppose \( p(x_i) \) does not increase in \( x_i \); then there must exist \( x_i < x_i < x_i \) such that \( p(x_i) > p(x_i) \). With a temporary abuse of notation, let \( \Pi(t; x_i) \) denote the incumbent’s expected utility if the incumbent’s cost is \( x_i \) and he chooses \( t \) as the bid-down-to level. On one hand, by the definition of \( p(x_i) \) and \( p(x_i) \), we have

\[
\Pi(p(x_i); x_i) - \Pi(p(x_i); x_i) \geq 0 \quad \text{and} \quad \Pi(p(x_i); x_i) - \Pi(p(x_i); x_i) \geq 0.
\]
On the other hand, we notice that
\[
\Pi(p(x_1^{(1)}); x_1^{(0)}) - \Pi(p(x_1^{(2)}); x_1^{(1)}) + \Pi(p(x_2^{(1)}); x_1^{(0)}) - \Pi(p(x_2^{(2)}); x_1^{(2)})
\]
\[
= \int_{p(x_1^{(1)})}^{p(x_1^{(2)})} \frac{d\Pi(t; x_1^{(1)})}{dt} dt - \int_{p(x_1^{(2)})}^{p(x_1^{(1)})} \frac{d\Pi(t; x_1^{(2)})}{dt} dt < 0,
\]
where the inequality holds because \(p(x_1^{(1)}) > p(x_1^{(2)})\) and \(d\Pi(t; x_1^{(1)})/dt > d\Pi(t; x_1^{(2)})/dt\). However, (5) contradicts (6), which implies that \(p(x_1)\) must increase in \(x_1\). Moreover, if \(\max(x_1^{(0)}, K/\beta) < p(x_1^{(0)}) < R\), then it must be
\[
\frac{d\Pi(t; x_1^{(1)})}{dt} \bigg|_{t=p(x_1^{(2)})} = 0;
\]
this implies that \(p(x_1^{(1)}) < p(x_1^{(2)})\); otherwise, if \(p(x_1^{(1)}) = p(x_1^{(2)})\), we have
\[
\frac{d\Pi(t; x_1^{(2)})}{dt} \bigg|_{t=p(x_1^{(2)})} > \frac{d\Pi(t; x_1^{(1)})}{dt} \bigg|_{t=p(x_1^{(2)})} = 0,
\]
contradicting \(p(x_1^{(2)})\)’s optimality.

Existence and uniqueness of \(x_2\). Given any \(x_i \in (K/\beta, R)\), we have \(d\Pi(t)/dt \mid_{t=x_1^{(1)}} = 0\) for all \(x_i \in (K/\beta, R)\), where (5) has the following two implications. Implication 1: \(p(x_i) > x_i\) for all \(x_i \in (K/\beta, R)\). Implication 2: \(p(x_i) > x_i\) for all \(x_i \in (K/\beta, R)\). To see that Implication 2 is true, note that, for any \(\delta > 0\), for all \(x_i \in (K/\beta, R)\) and all \(t \in [x_i, R]\), Equation (4) and the fact that \(U(\cdot)\) decreases together imply that \(d\Pi(t)/dt > E[R - \delta - K/\beta]U'(0) > 0\), which approaches \(E[R - K/\beta]U'(0) > 0\) as \(\delta\) approaches zero.

Implication 2 and the fact that \(p(x_i)\) is increasing together imply that there exists a unique threshold \(x_2 < R\) such that \(p(x_i) = R\) if and only if \(x_i > x_2\).

Existence and uniqueness of \(x_2\). The fact that \(U(\cdot)\) is concave implies that \(U(t-x_i) > 0\) if \(t > x_i\). Thus, when \(x_i > M + K/\beta\), there is a largest threshold \(x_2\) such that \(p(x_i) = \max(x_i, K/\beta)\) for all \(x_i \leq x_2\). This result is also true when \(R > M + K/\beta\). Note that (per (4)) \(d\Pi(t)/dt > 0\) for all \(R > M + K/\beta\). Thus, either \(p(x_i) = \max(x_i, K/\beta)\), or \(p(x_i) = R\). It is easy to check that \(\Pi(\max(x_i, K/\beta)) > \Pi(x_i)\) when \(x_i > x_2\).

Implication 1 implies that \(x_2 \leq K/\beta\), and hence \(p(x_i) = K/\beta\) for all \(x_i \leq x_2\). The fact that \(p(x_i)\) is increasing implies \(x_2 \leq x_i < R\) and that \(p(x_i) = K/\beta\) only if \(x_i > x_2\), i.e., \(x_2\) is unique.

Property of \(p(x_i)\) for \(x_i < x_2 < x_2\). When \(x_i < x_2 < x_2\), we have \(K/\beta < p(x_i) < R\), which is implied by the existence and the uniqueness of the thresholds \(x_2\) and \(x_2\). We have
\(p(x_i) > x_i\), which holds for \(x_i > K/\beta\) because of Implication 1 and holds for \(x_i < K/\beta\) (when \(x_i < K/\beta\) because \(p(x_i) > K/\beta \geq x_i\)). Finally, that \(p(x_i)\) strictly increases when \(x_i < x_2 < x_2\) because we have proved that \(p(x_i)\) strictly increases when \(\max(x_i, K/\beta) < p(x_i) < R\).

**Proof of Theorem 2**

**Effect of \(R\).** Consider any \(R_1 < R_2\) and any \(t_1 < t_2 < t_1\). For a given \(x_i\), when \(R = R_1\) we have \(\Pi(t_1) - \Pi(t_1) = \int_{t_1}^{t_2} d\Pi(t)/dt dt\), which does not change as \(R_1\) increases because \(d\Pi(t)/dt\) does not change with \(R\). This implies that the optimal bid-down-to levels when \(R = R_1\) and \(R_2\), denoted by \(p(x_i)_{R=R_1}\) and \(p(x_i)_{R=R_2}\) respectively, should be such that either \(p(x_i)_{R=R_1} = p(x_i)_{R=R_2}\) or \(p(x_i)_{R=R_1} > R_1 > p(x_i)_{R=R_2}\). Namely, the optimal bid-down-to level increases in \(R\), which in turn implies that the probability that the incumbent wins the auction directly decreases in \(R\).

**Effect of \(K\).** We prove it by showing that (i) \(p(x_i)\) decreases in \(K\) when \(p(x_i) > K/\beta\) and that (ii) \(p(x_i)_{R=R_k} = K/\beta\) for all \(K > \hat{K}\). The facts (i)-(ii) together imply that the probability that the incumbent wins the auction directly increases in \(K\).

If (i) is not true, then there must exist \(K_3 < K_2\) and \(t_1 < t_2\) such that either (or both) of the following two cases must be true.

**Case 1:** \(t_1 < t_2 < R\), \(\Pi(t_1) < \Pi(t_2)\), and \(p(x_i)_{R=R_1} < p(x_i)_{R=R_2}\). If (ii) is not true, then there must exist \(K_3 < K_2\) and \(t_1 < t_2\) such that either (or both) of the following two cases must be true.

**Case 2:** \(t_1 < K/\beta\), \(t_2 = R\), \(\Pi(t_1) > \Pi(t_2)\), and \(p(x_i)_{R=R_1} > p(x_i)_{R=R_2}\). If (ii) is not true, then there must exist \(K_3 < K_2\) and \(t_1 < t_2\) such that (or both) of the following two cases must be true.

**Case 3:** \(t_1 = K/\beta\), \(t_2 > K/\beta\), \(\Pi(t_1) < \Pi(t_2)\), and \(p(x_i)_{R=R_1} < p(x_i)_{R=R_2}\). Note that any of the four cases must imply that
\[
\Pi(t_2; K_2) - \Pi(t_2; K_3) < 0.
\]

Because the left-hand side of (7) equals \(\int_{K/\beta}^{K} [\Pi(t_1; K) - \Pi(t_1; K_2)] dt\), it suffices to draw a contradiction to prove (i) and (ii) by showing that \(\Pi(t_1; K) - \Pi(t_1; K_2)\) is decreasing in \(K\) for all \(K \in [K_1, K_2]\) under all four cases. We can rewrite (3) as
\[
\Pi(t_1; K_2) - \Pi(t_1; K_1) = \int_{[t_1, K/\beta]} U'(z + K/\beta - x) f(z) dz + \int_{[K/\beta, R]} U'(z + K/\beta - x) f(z) dz.
\]

To prove \(\Pi(t_1; K) - \Pi(t_1; K_2) < 0\) for all \(K \in [K_1, K_2]\) under all four cases, we show that
\[
\frac{d\Pi(t_1; K)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK}.
\]

\[
\frac{d\Pi(t_1; K)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK} = \frac{d\Pi(t_1; K_2)}{dK}.
\]
\[
\begin{align*}
&\geq 1 - \frac{1 - \beta}{\beta} \left[ F(t_1 - K, \beta) U'(t_1 - x_i) - F(t_2 - K, \beta) U'(t_2 - x_i) \right] \\
&\quad + \frac{1}{\beta} \int_{t_1 - K}^{t_2 - K} U'(x) \left( z + x - K \right) f_x(z) dz \\
&\quad - \frac{1}{\beta} \int_{t_1 - K}^{t_2 - K} U'(x) \left( z + x - K \right) f_x(z) dz \\
&\quad + \frac{1}{\beta} \int_{t_1 - K}^{t_2 - K} U'(x) \left( z + x - K \right) f_x(z) dz \geq 0.
\end{align*}
\]

The first equality is by (8), the second by integration by parts and \( U(\cdot) \) concave. We explain the first inequality for Case i-1; the other cases are similar. Note that Equation (4) and (d\( \Pi(t; K)/dt \)\)|\(_{t=0}\) = 0 implies that \( f_x(t_1 - K, \beta) \cdot [U(t_1 - x_i) - U(0)] = (1 - \beta) / \beta f_x(t_1 - K, \beta) U'(t_1 - x_i) \), which, together with the assumption that \( f_x(x)/f_x(x') \) increases in \( x \) implies that \( f_x(t_1 - K, \beta) [U(t_1 - x_i) - U(0)] \geq ((1 - \beta) / \beta) F_x(t_1 - K, \beta) U'(t_1 - x_i) \) for all \( K \in [K_1, K_2] \). Similarly, Equation (4), (d\( \Pi(t; K)/dt \)\)|\(_{t=0}\) = 0, and \( F_x(x)/f_x(x') \) increasing in \( x \) together imply \( f_x(t_1 - K, \beta) [U(t_2 - y_i) - U(0)] \leq ((1 - \beta) / \beta) F_x(t_2 - K, \beta) U'(t_2 - x_i) \) for all \( K \in [K_1, K_2] \).

**Effect of \( \beta \).** One can numerically check that the optimal bid-down-to level is not monotone in \( \beta \), e.g., when \( K = 0.24 \), \( \beta \in [0.4, 0.9] \), \( F_r \sim U[0.1] \), \( F_r \sim U[0.1] \), and the incumbent is risk neutral. □

**Proof of Theorem 3**

For any \( 0 < \beta < 1 \), the expected total cost under prequalification linearly increases in qualification cost \( K \) (per (1)), whereas the expected total cost under postqualification is bounded above by \( R \) (per (2)). Thus, postqualification yields a lower expected total cost when \( K \) is large enough; namely, there must exist a \( K \geq 0 \) such that the buyer prefers postqualification whenever \( K > K \).

As \( \beta \) approaches \( K/R \), the buyer’s expected total cost approaches \( (K/R)E[\max[x, \min[R, x_4]]] + R \) under prequalification (per (1)) and approaches \( R \) under postqualification (per (2)), which implies that \( K \) approaches zero. As \( \beta \) approaches 1, the buyer’s expected total cost approaches \( E[\max[x, \min[R, x_4]]] + K \) under prequalification (per (1)) and approaches \( E[\max[x, \min[x + K, R]]] \) under postqualification (per (2)). This implies that \( K \) approaches zero because it is easy to show that \( E[\max[x, \min[R, x_4]]] + K > E[\max[x, \min[x + K, R]]] \) for all \( K > 0 \). □

**Optimal Bid Functions for Figure 1: Risk-Neutral Incumbent Facing Uniform \( F_r \).** When \( U(t - x_i) = t - x_i \) and \( F_r \sim U[0, 100] \), per (3) and (4), we have for \( t \in [K/\beta, R] \),

\[
\Pi(t) = \begin{cases} \\
\left[ \frac{\int_0^R (y - x_i) dy + (1 - \beta)(t - x_i)}{100} \right] / 100, & \text{if } 100 + K/\beta \geq R; \\
\left[ \frac{\int_0^{100 + K/\beta} (y - x_i) dy + (1 - \beta)(t - x_i)}{100} \right] / 100, & \text{if } 100 + K/\beta < R \text{ and } t \leq 100 + K/\beta; \\
(1 - \beta)(t - x_i), & \text{if } 100 + K/\beta < R \text{ and } t > 100 + K/\beta.
\end{cases}
\]

and

\[
\frac{d\Pi(t)}{dt} = \begin{cases} \\
\left[ \beta x_i + (1 - 2\beta)t - \frac{K(1 - \beta - \beta)}{\beta} \right] / 100, & \text{if } 100 + K/\beta \geq R, \text{ or}; \\
\text{if } 100 + K/\beta < R \text{ and } t \leq 100 + K/\beta; \\
1 - \beta, & \text{if } 100 + K/\beta < R \text{ and } t > 100 + K/\beta.
\end{cases}
\]

Let \( t^*(x) = (\beta x_i) / (2\beta - 1) - ((1 - \beta)K/\beta(2\beta - 1)) \), i.e.,

\[
(d\Pi(t)/dt\)|_{t=t^*(x)} = 0 \text{ if } 100 + K/\beta \geq R, \text{ or if } 100 + K/\beta < R \text{ and } t \leq 100 + K/\beta.
\]

**Cases with \( R \leq 100 + K/\beta \).** Note that \( \Pi(t) \) is convex in \( t \) when \( 0 < \beta < 1/2 \), is linear in \( t \) when \( \beta = 1/2 \), and is concave in \( t \) when \( 1/2 < \beta < 1 \). When \( 0 < \beta < 1/2 \), the convexity (actually, linearity when \( \beta = 1/2 \)) of \( \Pi(t) \) implies that the optimal solution \( p(x) \) equals either \( t = \max[x, K/\beta] \) or \( t = R \); this together with the fact that \( p(x_i) > x_i \) for all \( x_i \in (K/\beta, R) \) (shown in the proof of Theorem 1) implies that the optimal solution \( p(x) \) equals either \( t = K/\beta \) or \( t = R \). Note that \( \Pi(R) = [(R - K/\beta)(1 - \beta)(R - x_i) + (R_i - x_i)] \cdot (100 + K/\beta - R)/100 + K(1 - \beta)K/\beta) \cdot (100 + K - R) /100 \). It is easy to check that \( \Pi(t) > \Pi(K/\beta) \) if and only if \( x_i > K/2\beta - R/2 \beta + R \). Namely, \( x_w = x_b = K/2\beta - R/2 \beta + R \). When \( 1/2 < \beta < 1 \), the concavity of \( \Pi(t) \) implies that \( p(x) = \max[K/\beta, \min(t^*(x), R)] \). That is, \( p(x) = K/\beta \) if \( x_i \leq K/\beta \) (because \( t^*(x) < K/\beta \) when \( x_i < K/\beta \), \( p(x) = R \) if \( x_i \leq (1 - \beta)K/\beta + (1 - \beta)K/\beta \) (because \( t^*(x) \geq R \) when \( x_i \geq (1 - \beta)K/\beta + (1 - \beta)K/\beta \), and \( p(x) = t^*(x) \) if \( K/\beta < x_i < (1 - \beta)K/\beta + (1 - \beta)K/\beta \).

**Cases with \( R > 100 + K/\beta \).** When \( 0 < \beta \leq 1/2 \), the convexity of \( \Pi(t) \) over \( t \in [K/\beta, 100 + K/\beta] \) and the fact that \( \Pi(t) \) increases when \( t \in [100 + K/\beta, R] \) together imply that \( \Pi(t) \) is quasiconvex, and hence the optimal solution \( p(x) \) equals either \( t = \max[x, K/\beta] \) or \( t = R \). Again, the fact that \( p(x_i) > x_i \) for all \( x_i \in (K/\beta, R) \) (shown in the proof of Theorem 1) further implies that the optimal solution \( p(x) \) equals either \( t = K/\beta \) or \( t = R \). It is easy to check that \( \Pi(R) = (1 - \beta)(R - x_i) \) and \( K(1 - \beta)K/\beta = (100 + K/\beta - R) /100 + K(1 - \beta)K/\beta) \cdot (100 + K - R) /100 \). Namely, \( x_w = x_b = K/2\beta + 50/\beta - R/\beta + R \). When \( 1/2 < \beta < 1 \), the concavity of \( \Pi(t) \) together imply that \( \Pi(t) \) is quasiconcave, and hence the optimal solution \( p(x) \) equals either \( t = \max[x, K/\beta] \) or \( t = R \).
\( x_t = \hat{x}_t \). In particular, \( \hat{x}_t \) solves \( \int_{\hat{x}_t}^{100+K/\beta} (y - \hat{x}_t) \, dy + \left[ \hat{f}(\hat{x}_t) - \frac{K/\beta}{1 - \beta} \hat{f}(\hat{x}_t) \right] / 100 = \Pi(\hat{x}_t) - \Pi(1 - \beta) \cdot (1 - \hat{x}_t) \); after simplification, we have for given \( K, \beta, \) and \( R, x_t = \hat{x}(K, \beta, R) \) is the \( x_t \in (-\infty, ((2\beta - 1)100)/\beta + K/\beta) \)
solving the following equation:

\[
0 = \begin{cases}
\frac{(\beta x_t - K)^2}{200(2\beta - 1)} - \beta x_t + \frac{K}{\beta} + 50 - (1 - \beta) R, \\
\frac{K}{\beta} < x_t \leq \frac{(2\beta - 1)100}{\beta} + \frac{K}{\beta'}, \\
-\beta x_t + \frac{K}{\beta} + 50 - (1 - \beta) R, & \text{if } x_t \leq \frac{K}{\beta}.
\end{cases}
\]

Finally, if \( \hat{x}(K, \beta, R) > K/\beta \), we have \( p(x_t) = \hat{f}(x_t) \) for \( x_t < \hat{x}(K, \beta, R) \) (i.e., \( p(x_t) = \hat{f}(x_t) \) for \( K/\beta < x_t < \hat{x}(K, \beta, R) \) and \( p(x_t) = \hat{f}(x_t) \) for \( x_t \leq K/\beta \)), which implies that \( x_W = K/\beta \); if \( \hat{x}(K, \beta, R) \leq K/\beta \), we have \( p(x_t) = \hat{f}(x_t) \), which implies that \( x_W = x_t \). To summarize, \( x_W = \min[K/\beta, x_t] \).

References


